## Overview of this week's lectures:

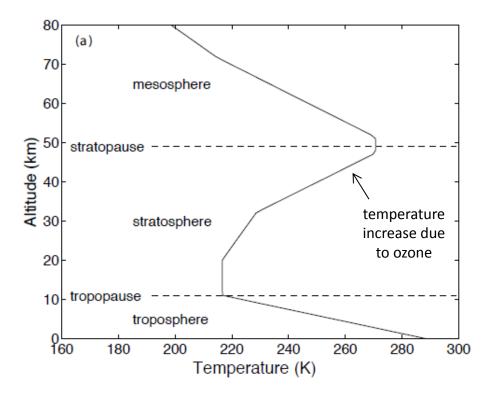
- Global circulation and transport
- Satellite observations of stratospheric temperature and water vapor
- Global upper troposphere lower stratosphere (UTLS)
- Circulation and transport near the tropical tropopause layer (TTL)
- UTLS monsoon circulations and water vapor isotopes
- Research seminar: tropical dynamics with GPS radio occultation data

Lecture 1: Global atmospheric circulation and satellite observations

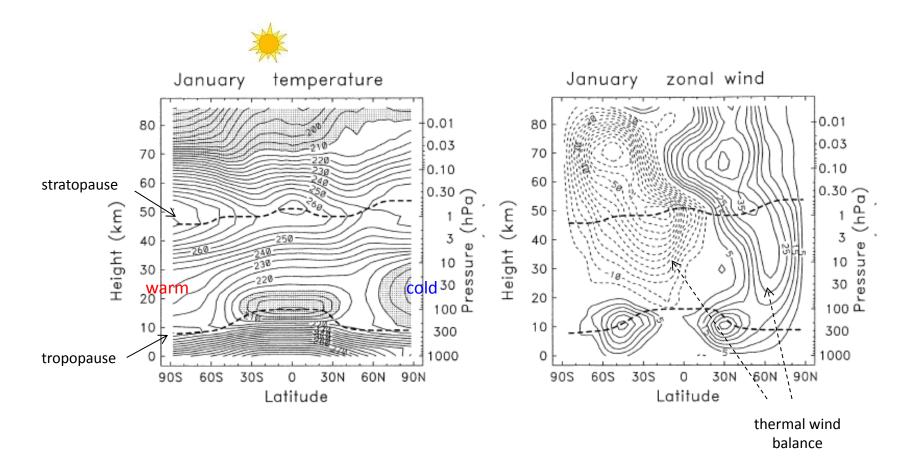
• large-scale circulation of the troposphere and stratosphere

- Climatology and variability of the stratosphere
- wave forcing of the zonal mean flow
- Rossby waves: propagation and dissipation (critical layers)
- Tropospheric baroclinic wave life cycles
- large-scale tropical circulations
- QBO and ENSO
- Large-scale transport

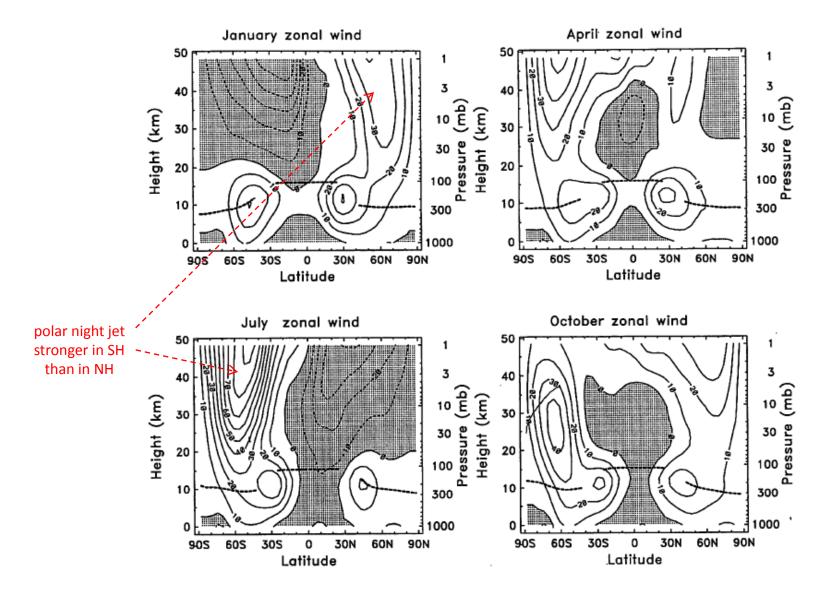
## The Standard Atmosphere



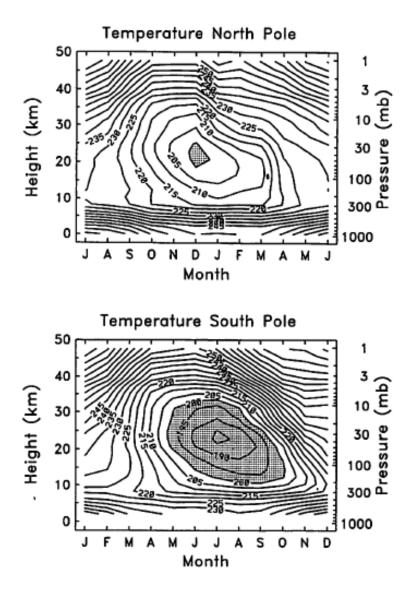
#### Climatological temperatures and zonal winds in January

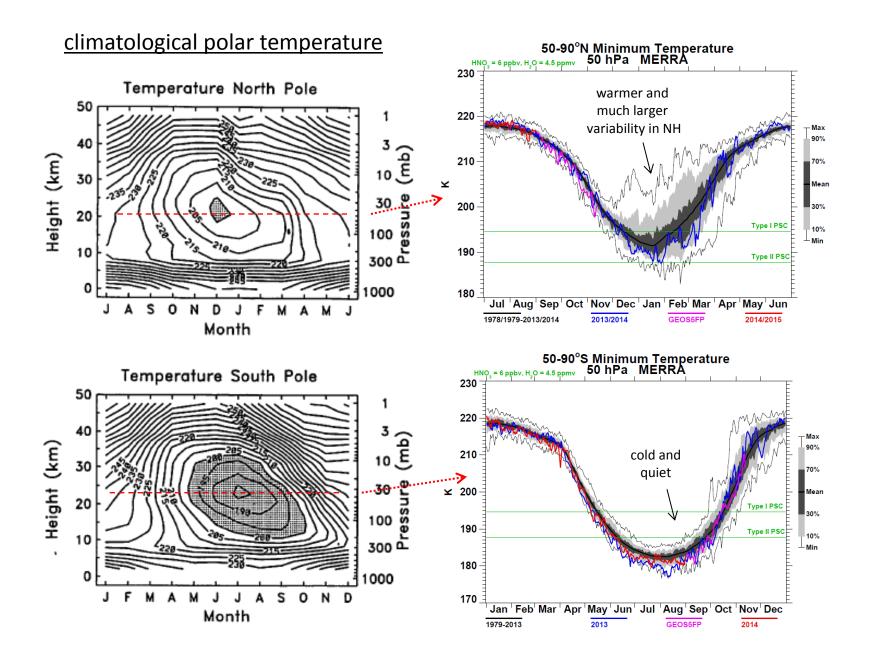


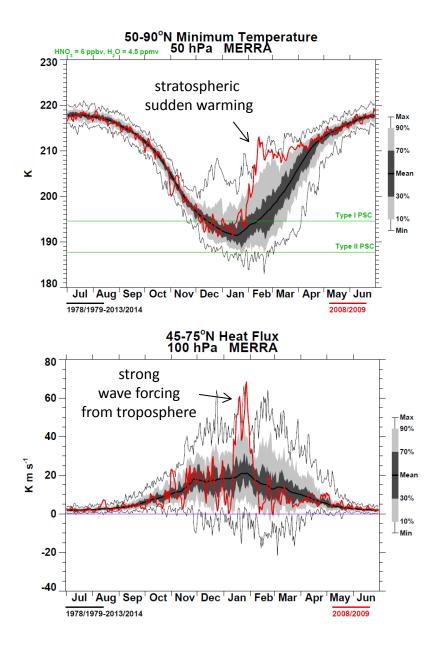
#### Seasonal cycle of zonal mean zonal winds



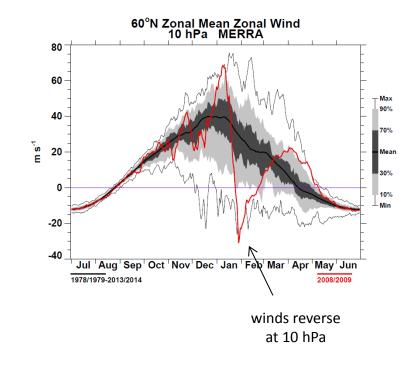
## climatological polar temperature







- Variability in NH winter stratosphere tied to large-scale forcing from troposphere.
- Episodic forcing produces 'stratospheric sudden warming' events.
- Largest observed stratosphere sudden warming in January 2009

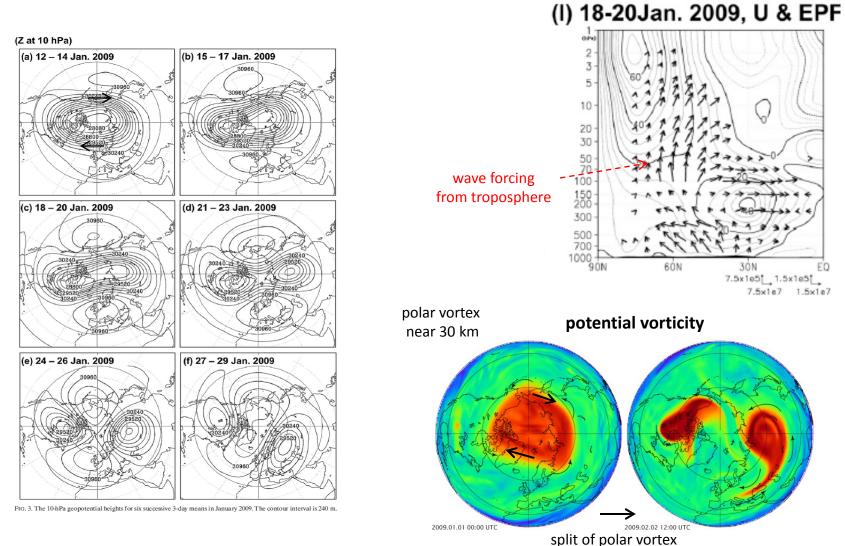


#### A Major Stratospheric Sudden Warming Event in January 2009

YAYOI HARADA, ATSUSHI GOTO, HIROSHI HASEGAWA, AND NORIHISA FUJIKAWA

Climate Prediction Division, Japan Meteorological Agency, Tokyo, Japan

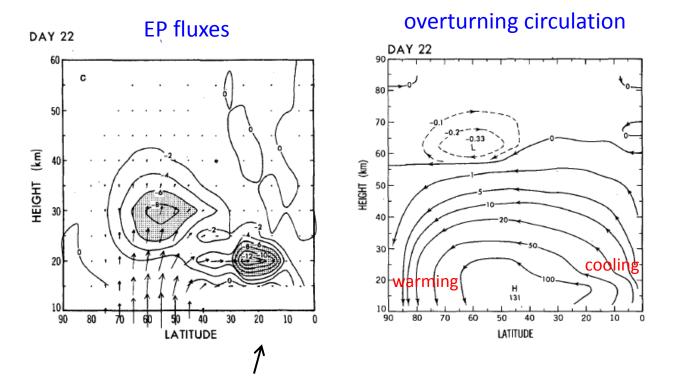
#### JAS 2010



EP flux

#### A Dynamical Model of the Stratospheric Sudden Warming

TAROH MATSUNO<sup>1</sup> Geophysical Fluid Dynamics Laboratory, NOAA, Princeton University, Princeton, N. J. (Manuscript received 29 March 1971, in revised form 16 August 1971) solution to puzzle of stratospheric warmings



#### Some Eulerian and Lagrangian Diagnostics for a Model Stratospheric Warming<sup>1</sup>

T. DUNKERTON, C.-P. F. HSU<sup>2</sup> AND M. E. MCINTYRE<sup>3</sup>

Department of Atmospheric Sciences, University of Washington, Seattle 98195

(Manuscript received 30 May 1980, in final form 11 December 1980)

## <u>Governing equations for the zonal mean flow</u> (Transformed Eulerian mean)

EP flux divergence (wave forcing)

zonal momentum balance
$$\frac{\partial \overline{u}}{\partial t} - \hat{f}\overline{v}^* = DF^{\prime}$$
thermodynamic balance $\frac{\partial \overline{T}}{\partial t} + \overline{v}^* \frac{1}{a} \frac{\partial \overline{T}}{\partial \phi} + \overline{w}^* S = \overline{Q},^{\prime}$ continuity equation $(a \cos \phi)^{-1} \frac{\partial}{\partial \phi} (\overline{v}^* \cos \phi) + e^{z/H} \frac{\partial}{\partial z} (\overline{w}^* e^{-z/H}) = 0,$ geostrophic thermal wind $f \frac{\partial \overline{u}}{\partial z} + \frac{R}{aH} \frac{\partial \overline{T}}{\partial \phi} = 0.$ 

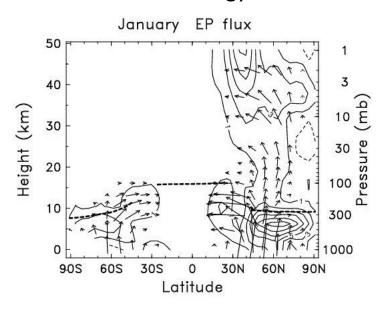
Andrews et al, 1987

#### EP flux divergence (wave forcing)

#### Eliassen-Palm fluxes:

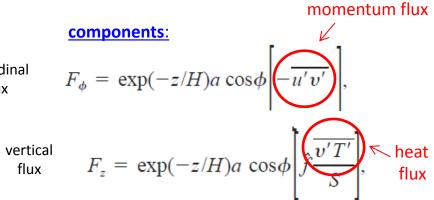


#### climatology



#### components:

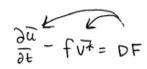
latitudinal flux



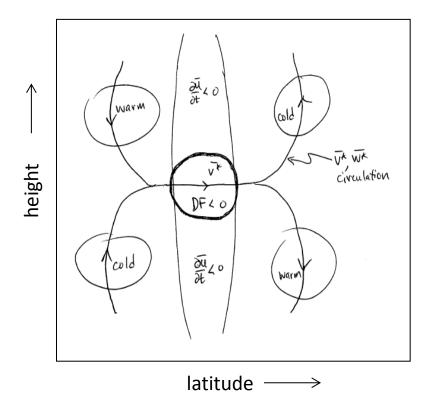
#### **Important points:**

- DF quantifies zonal momentum forcing
- **F** proportional to 'wave activity' flux (DF shows sources and sinks of waves)
- $F_{\phi}$  and  $F_{z}$  indicate direction of wave propagation

Response of a balanced vortex to localized EP flux forcing (DF)

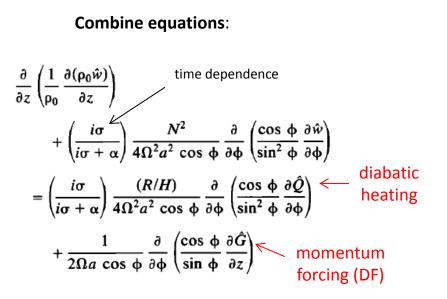


- response is balanced
   between <sup>d</sup>𝔅<sub>t</sub> and f 𝔽
  - V#, w\* and dy act to extend DF forcing non-locally
  - · overall circulation maintains thermal wind balance



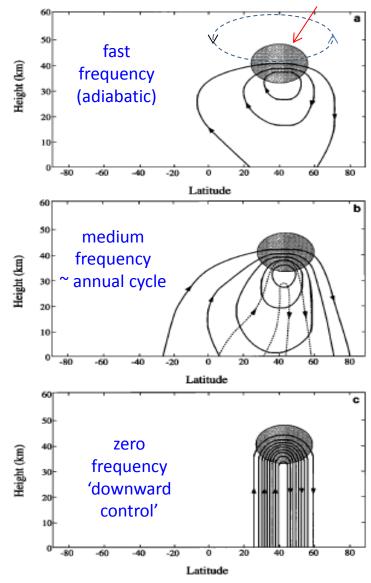
#### Circulation response depends on frequency of forcing:

momentum forcing (DF)

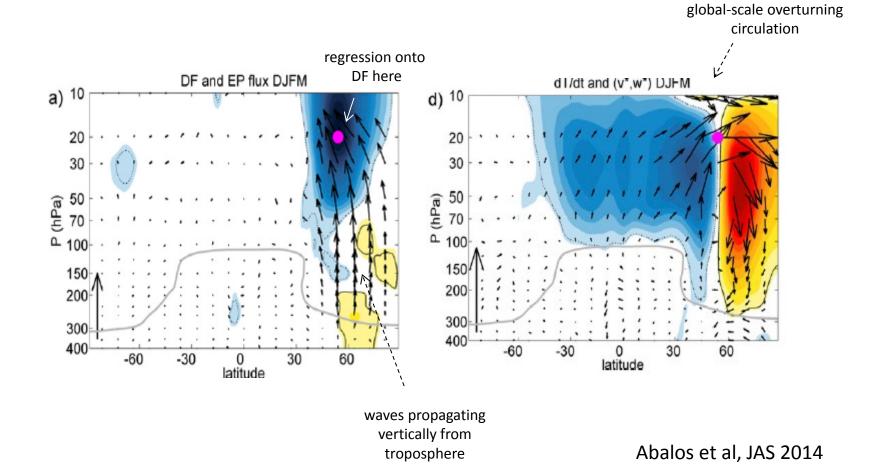


In general both Q and DF drive the mean circulation. These plots show the response to isolated forcing from Rossby wave EP flux divergence. The lower cell becomes more important for slower forcing.

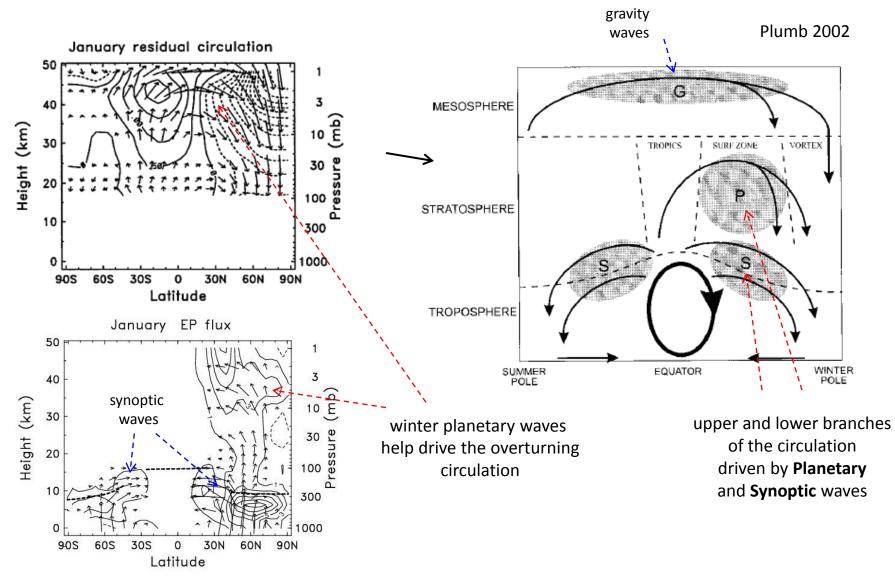
> Haynes et al 1991 Holton et al 1995



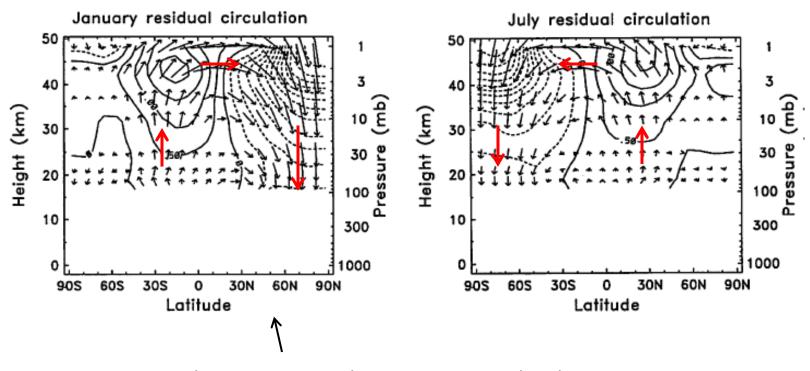
### Wave - mean flow patterns from observations (reanalysis data)



# Climatology of stratospheric overturning circulation

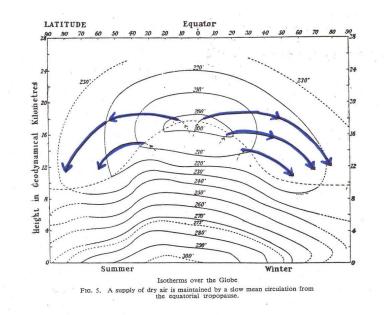


The overturning circulation reverses between solstice seasons



circulation is stronger during NH winter, related to stronger wave forcing from troposphere The stratospheric overturning circulation is often termed the Brewer-Dobson circulation (closely related to the Lagrangian or transport circulation)

deduced by Brewer (1949) studying stratospheric water vapor and Dobson (1956) studying stratospheric ozone



see recent review by Butchart 2014

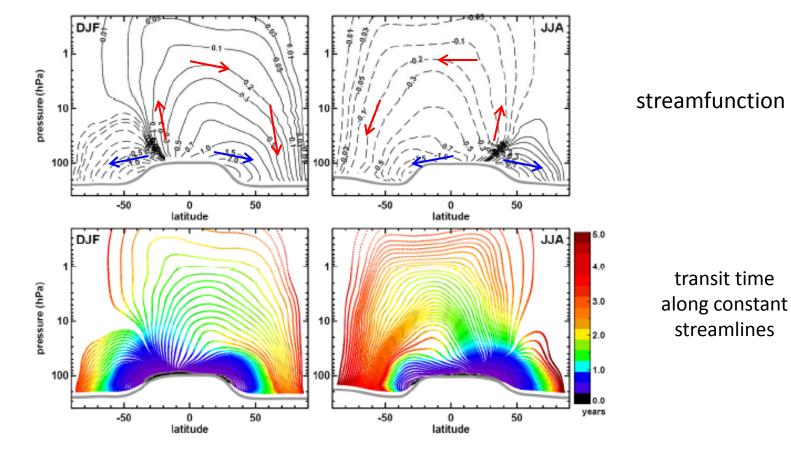
# Residual circulation trajectories and transit times into the extratropical lowermost stratosphere

T. Birner  $^{1}$  and H. Bönisch  $^{2}$ 

ACP 2011

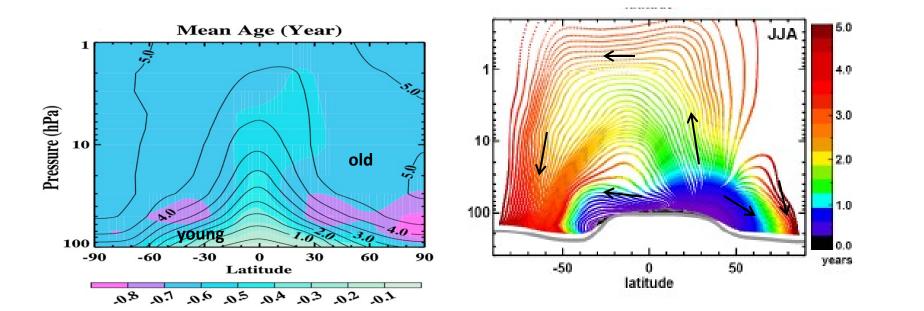
renewed appreciation that there are upper and lower branches of the BDC

Red: deep branch (slow) Blue: shallow branch (fast)



Transit time is closely related to 'mean age' (time since air entered stratosphere)

transit time



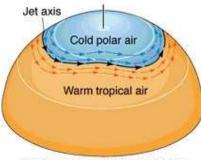
Air at any particular location is characterized by a distribution of transit times and ages (so-called age spectrum)

Birner and Bönisch, 2011, Atmos. Chem. Phys.

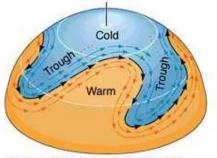
### Key points:

- Asymmetry in winter stratosphere circulations: more disturbed in the NH, cold and quiet in the SH
- The stratosphere is forced by waves from the tropopshere (stronger forcing in NH; episodic stratospheric sudden warmings)
- Dynamical response of balanced vortex to wave forcing (non-local temperature and wind changes)
- Eliassen-Palm (EP) fluxes quantify wave forcing
- Brewer-Dobson transport circulation (deep and shallow branches)

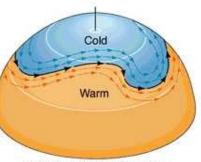
## Rossby waves



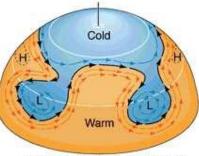
The jet stream begins to undulate.



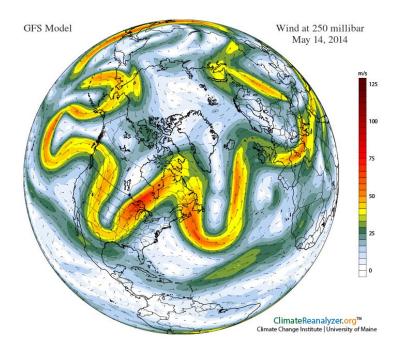
Waves are strongly developed. The cold air occupies troughs of low pressure. Copyright © A.N. Strahler.



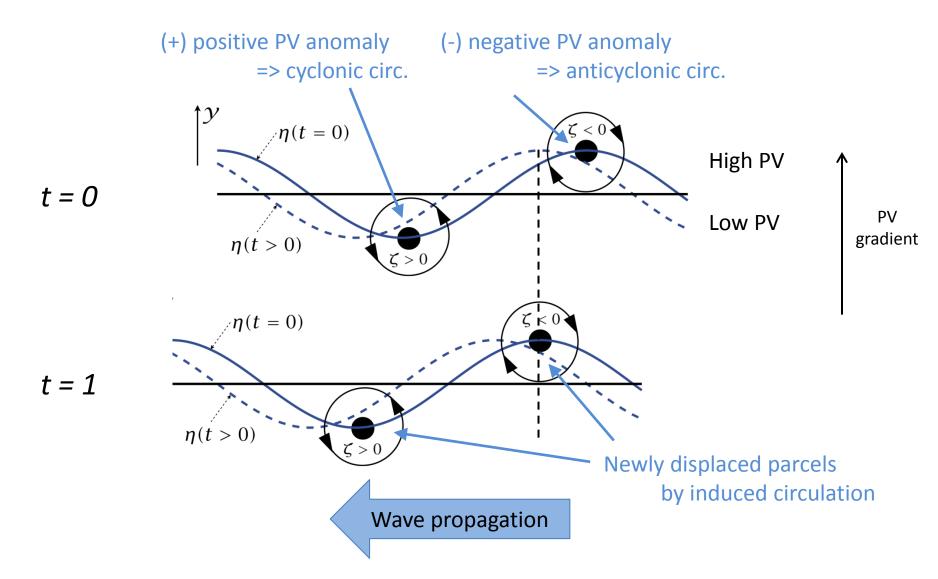
Rossby waves begin to form.



When the waves are pinched off, they form cyclones of cold air.



## Rossby wave (potential vorticity wave)



Vallis 2006 (Atmospheric and Oceanic Fluid Dynamics, 2<sup>nd</sup> Ed.)

## Rossby wave propagation: quasi-geostrophic linearized PV equation

$$\begin{pmatrix} \frac{\partial}{\partial t} + \frac{\bar{u}}{a\cos\phi} \frac{\partial}{\partial\lambda} \end{pmatrix} q'_{(M)} + a^{-1} \bar{q}_{\phi} v' = 0,$$
  
where  $v' = 0,$   
eddy PV background PV gradient  
 $\psi'$   
form:  
 $\bar{q}_{\phi} = 2\Omega \cos\phi - \left[ \frac{(\bar{u}\cos\phi)_{\phi}}{a\cos\phi} \right]_{\phi} - \frac{a}{\rho_0} \left( \frac{\rho_0 f^2}{N^2} \bar{u}_z \right)_z.$   
 $\bar{q}_z = 2\Omega \cos\phi - \left[ \frac{(\bar{u}\cos\phi)_{\phi}}{a\cos\phi} \right]_{\phi} - \frac{a}{\rho_0} \left( \frac{\rho_0 f^2}{N^2} \bar{u}_z \right)_z.$ 

$$\frac{f^2}{a^2 \cos \phi} \left( \frac{\cos \phi}{f^2} \Psi_{\phi} \right)_{\phi} + \frac{f^2}{N^2} \Psi_{zz} + n_s^2 \Psi = 0$$

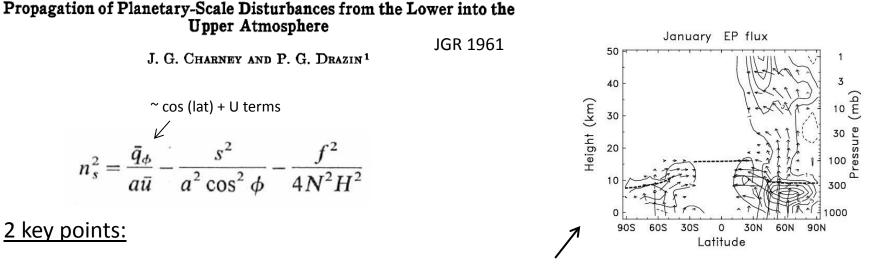
wave equation: propagation for  $n_{s}^{2} > 0$ 

$$n_s^2 = \frac{\bar{q}_{\phi}}{a\bar{u}} - \frac{s^2}{a^2 \cos^2 \phi} - \frac{f^2}{4N^2 H^2}$$

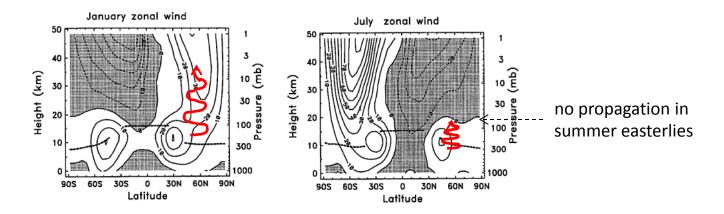
refractive index

wave solution

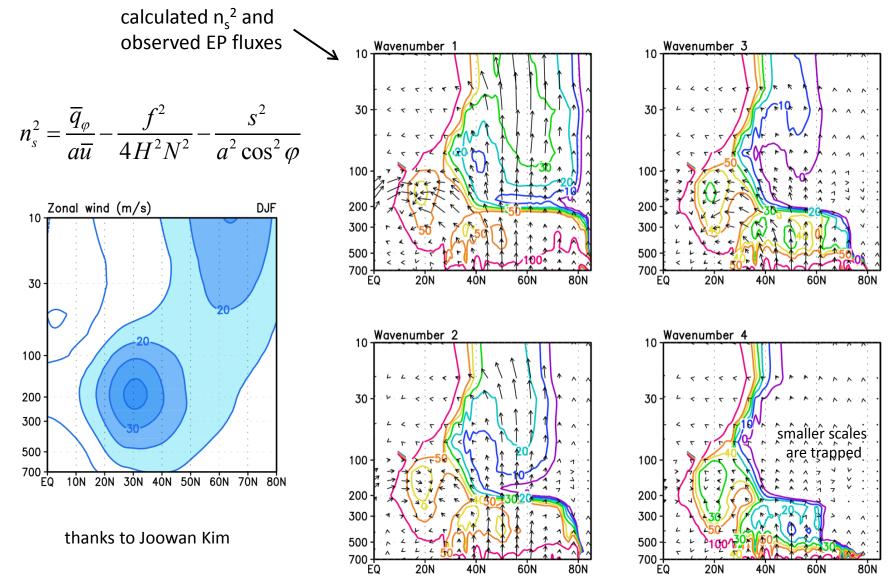
 $\Phi' = e^z$ 



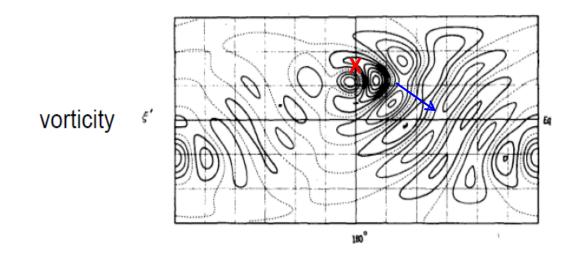
- n<sub>s</sub><sup>2</sup> proportional to ~cos (lat) (Rossby wave refraction towards low latitudes)
- vertical propagation for U > 0 and small zonal wavenumbers (planetary waves propagate to stratosphere only during winter)



# zonal wave 1 and 2 can propagate vertically in climatological basic state

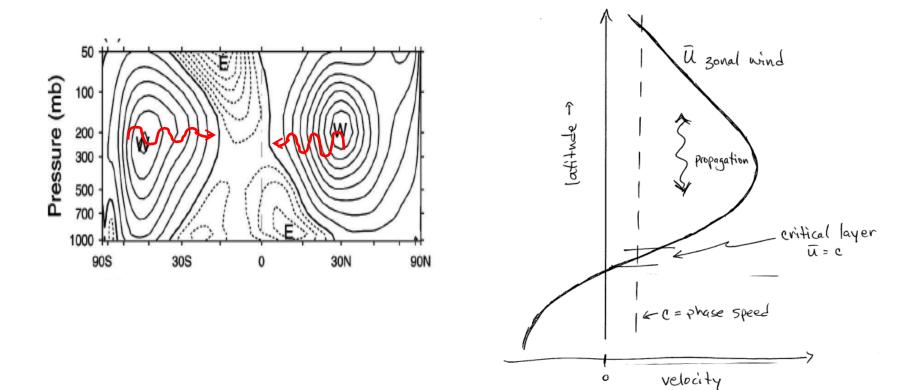


Rossby wave dispersion on the sphere: response to isolated topography

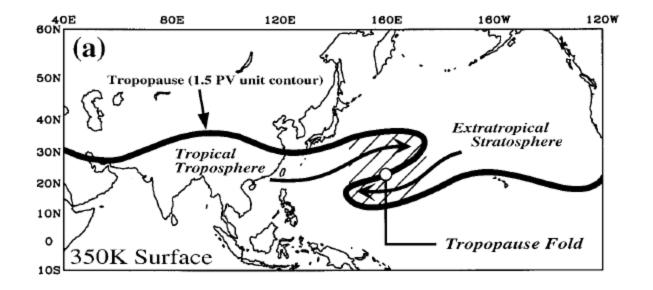


Steady state solution of the shallow water equations linearized about a <u>constant angular velocity flow</u> and perturbed by a circular mountain at 30°N, 180°E. The model includes linear drag. After B. J. Hoskins, *Horizontal Wave Propagation on the Sphere*. in *The General Circulation*. *Theory, Modeling and Observations*. NCAR Summer Colloquium 1978.



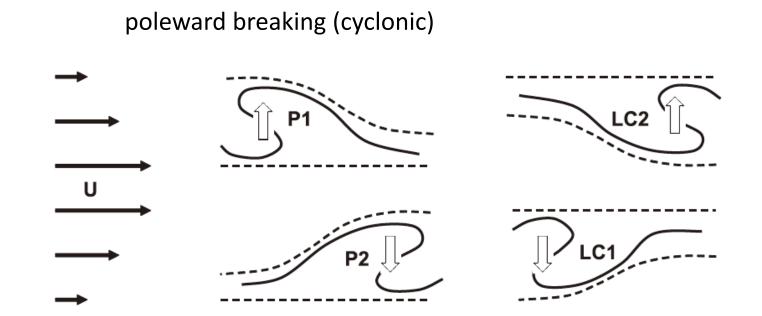


### Breaking Rossby waves: overturning of PV contours





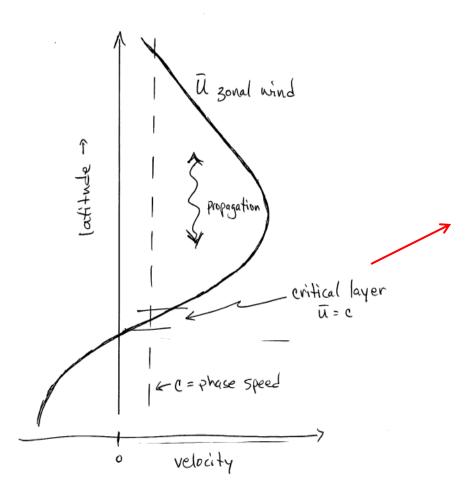
Postel and Hitchman 1999 Homeyer et al 2013 Two types of wave breaking, depending on shear of background winds



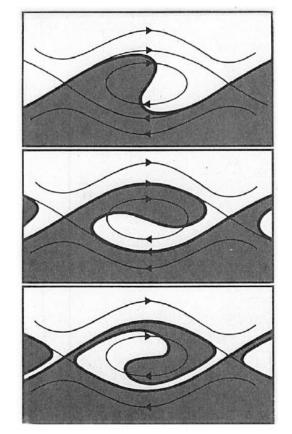
equatorward breaking (anticyclonic)

Gabriel and Peters 2008

Rossby wave critical layer interactions (critical layer: U = c)

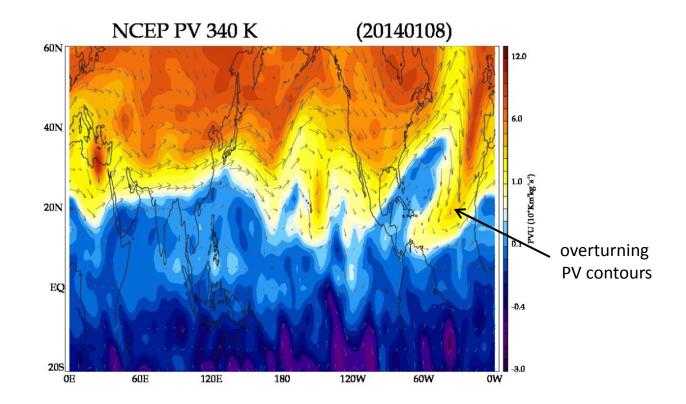


nonlinear overturning at critical layer (irreversible transport and mixing)

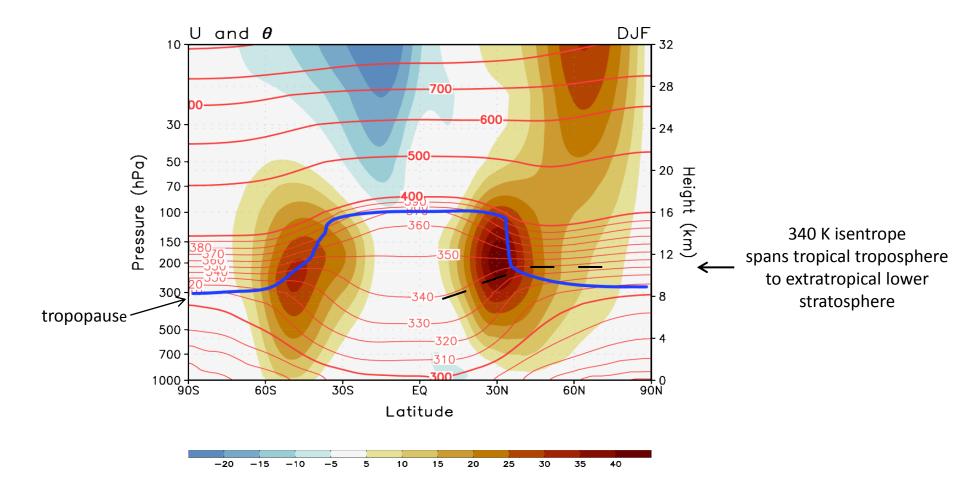


Warn and Warn 1978

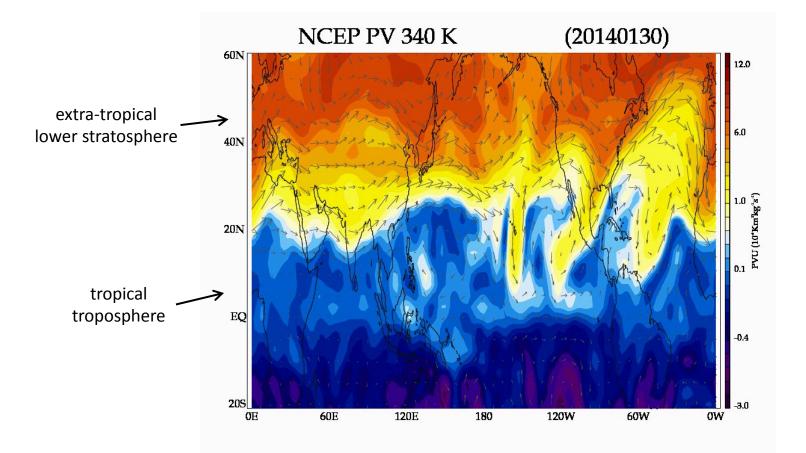
Example of a large-scale breaking Rossby wave



### fast, synoptic flow mainly along isentropes:

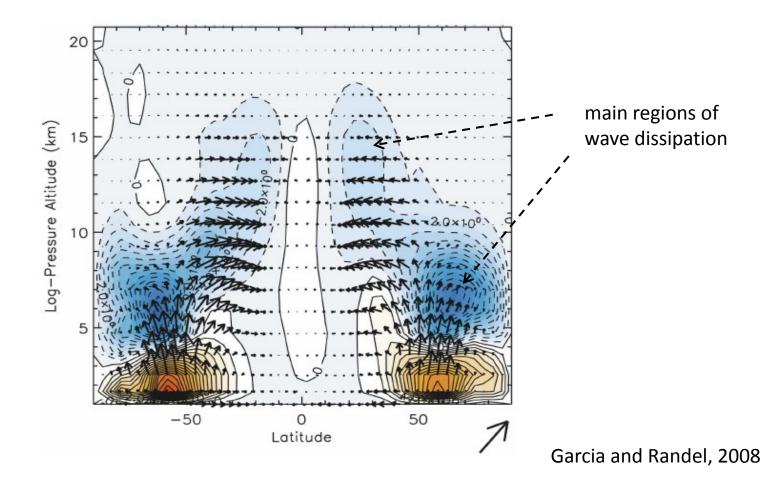


## Rossby waves during January-May at 340 K

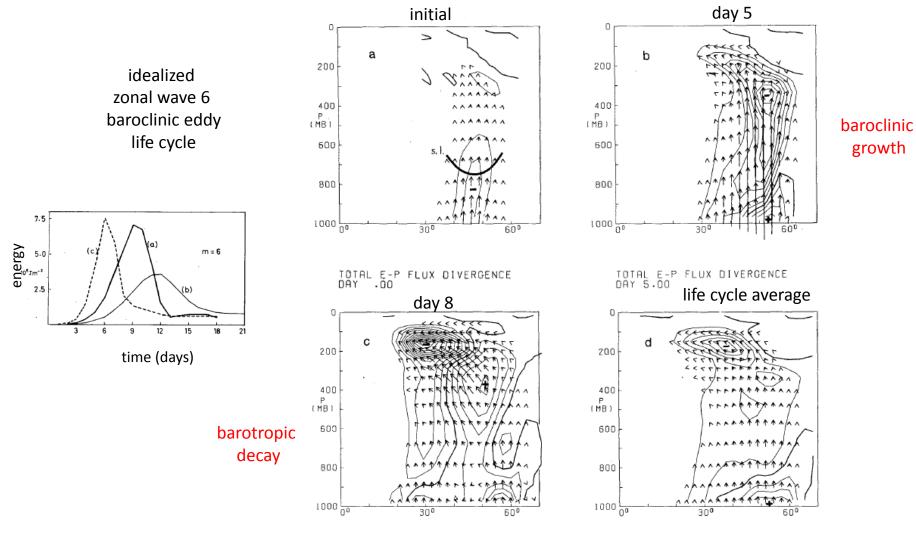


Key points:

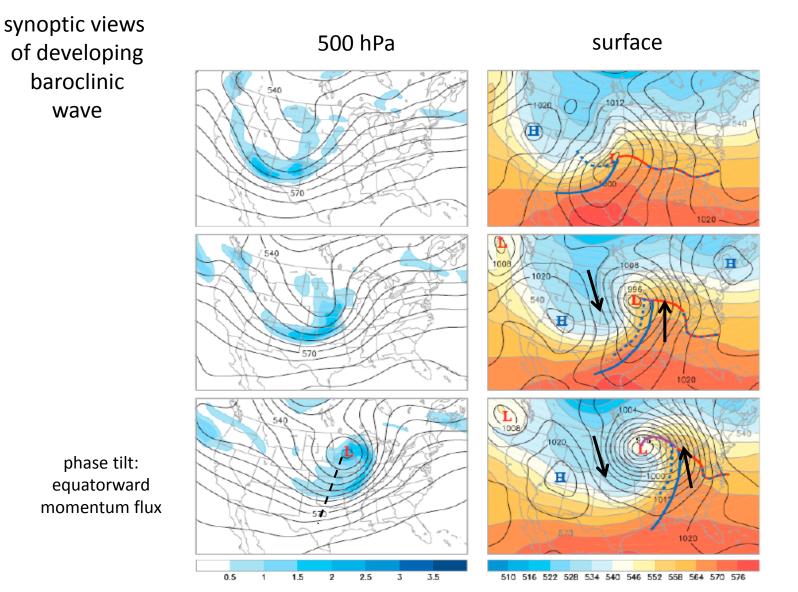
- Rossby waves: conserve PV
- General refraction of Rossby waves towards low latitudes
- Latitudinal or vertical propagation for U > 0 (more generally U > c)
- Rossby wave breaking near critical lines (U = c)
- Poleward or equatorward breaking depending on background U shear
- Key mechanism for dissipation, transporting PV or trace species



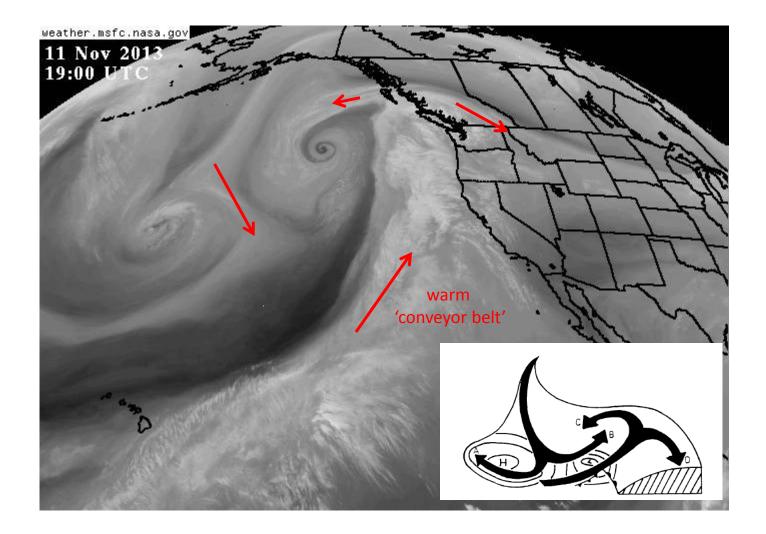
## Extratropical EP flux patterns are related to baroclinic wave life cycles



Simmons and Hoskins 1980 Edmon et al 1980



poleward heat flux

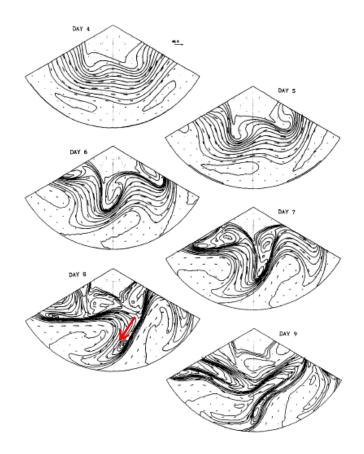


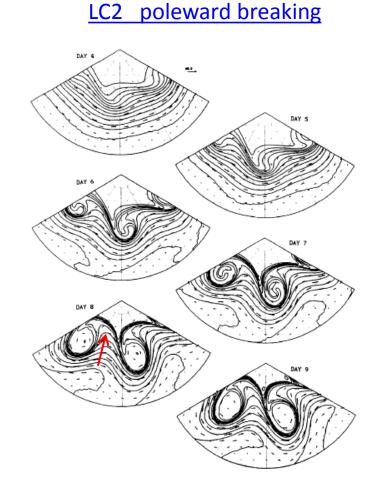
Q. J. R. Meteorol. Soc. (1993), 119, pp. 17-55

#### Two paradigms of baroclinic-wave life-cycle behaviour

By C. D. THORNCROFT<sup>1\*</sup>, B. J. HOSKINS<sup>1</sup> and M. E. McINTYRE<sup>2</sup> <sup>1</sup>Department of Meteorology, University of Reading <sup>2</sup>Department of Applied Mathematics and Theoretical Physics, University of Cambridge

# LC1 equatorward breaking

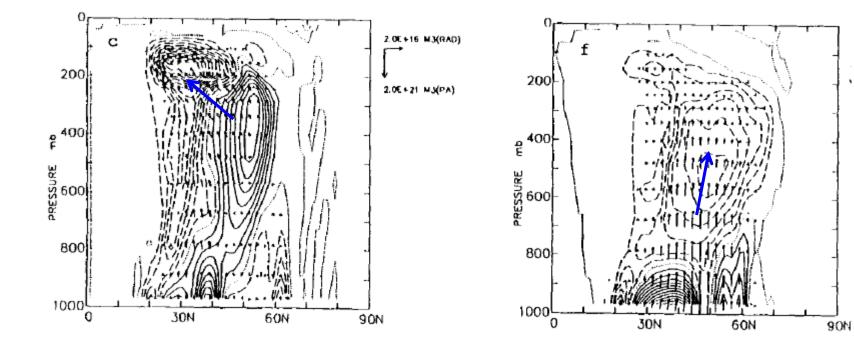




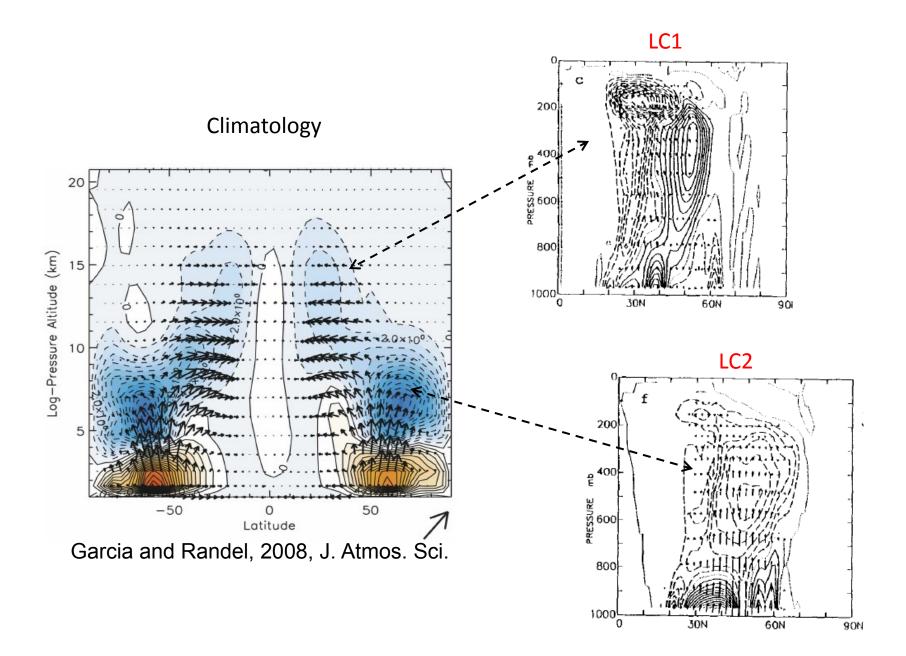
Idealized baroclinic wave life cycles

## equatorward propagation (LC1)

## poleward propagation (LC2)

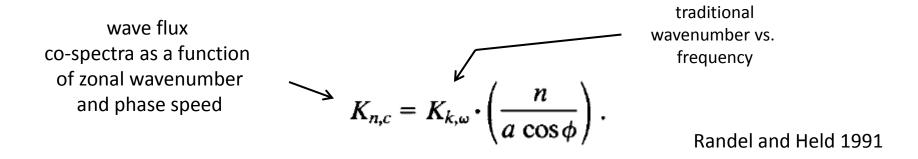


Thorncroft et al., 1993, Q.J.R. Meteorol. Soc.

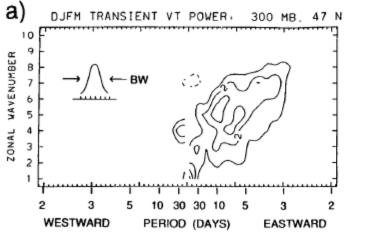


Thorncroft et al., 1993, Q.J.R. Meteorol. Soc.

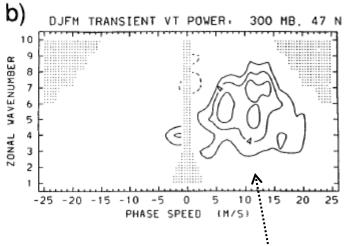
#### Using phase speed spectra to diagnose critical layer interactions



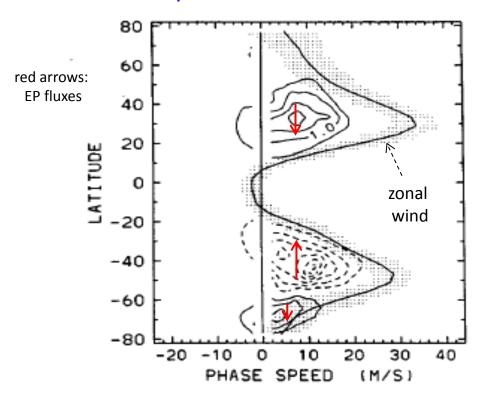
#### wavenumber vs. frequency



#### wavenumber vs. phase speed



Rossby waves move eastward at ~5-15 m/s Integrate over wavenumber to derive eddy flux phase speed spectra

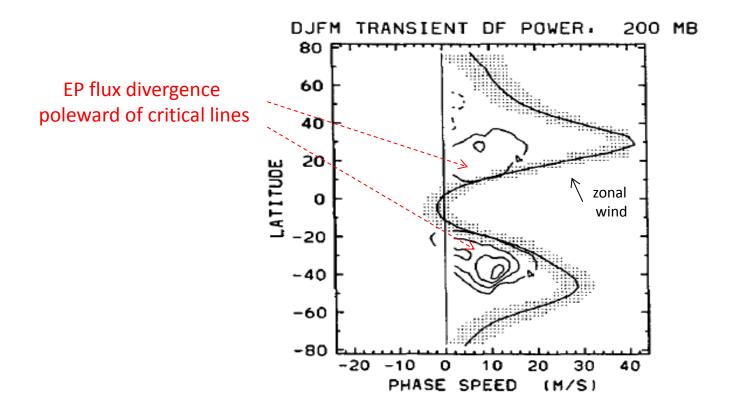


## eddy momentum flux u'v' 300 hPa

- EP fluxes: propagation to near critical lines (c = U)
- evidence for critical layer behavior

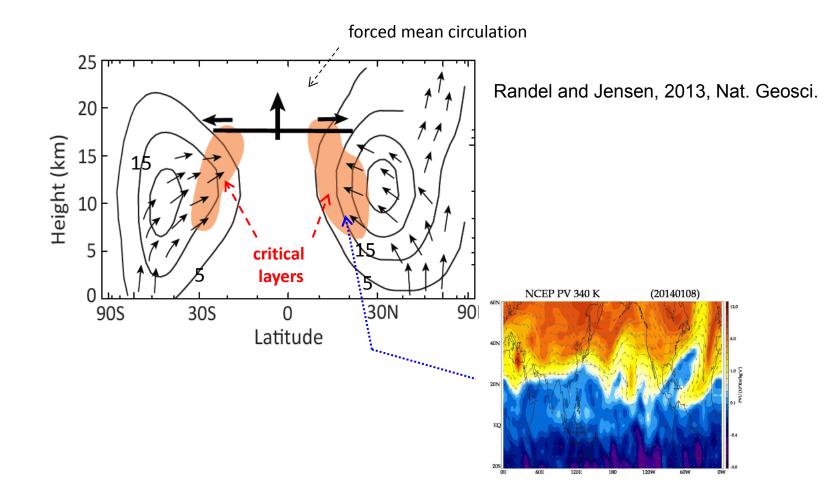
Randel and Held 1991

EP flux divergence phase speed spectra



Randel and Held 1991

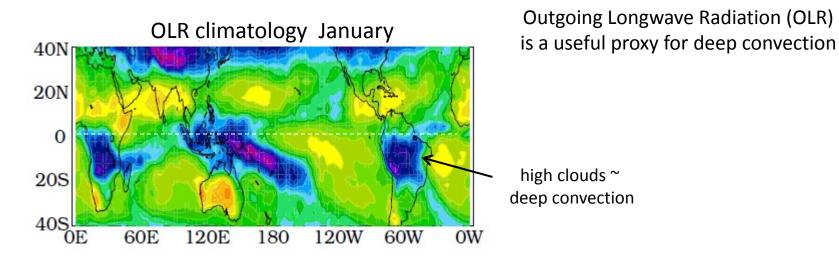
Subtropical critical layers for Rossby waves with phase speeds ~ 5-15 m/s

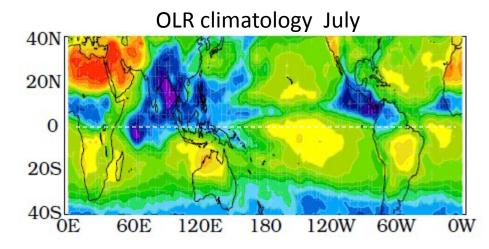


## Key points:

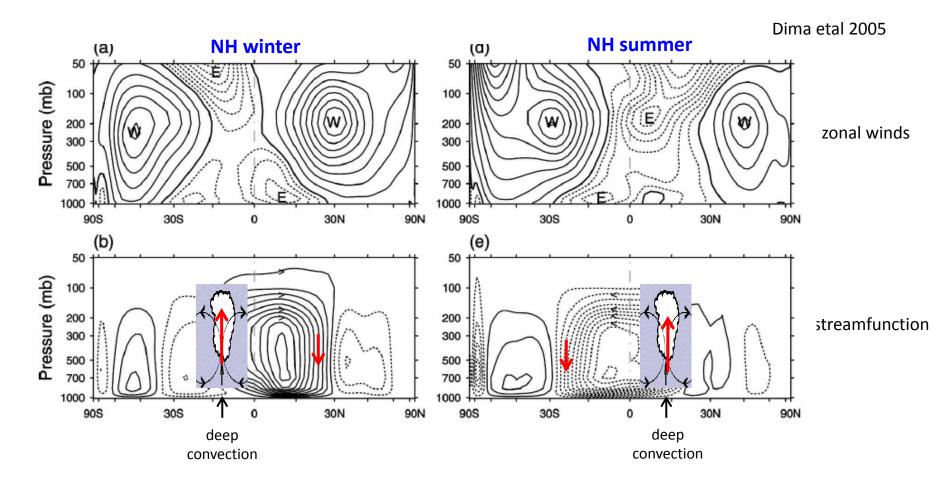
- Baroclinic wave life cycles: baroclinic growth and barotropic decay
- Two idealized types of life cycles: equatorward and poleward wave breaking (LC1 and LC2)
- Consistent with tropospheric EP flux circulation statistics
- Phase speed spectra: clear evidence for critical layers in subtropics (important influence of extratropical waves on tropical circulations)

# Large-scale tropical circulations are forced by latent heating from deep convection

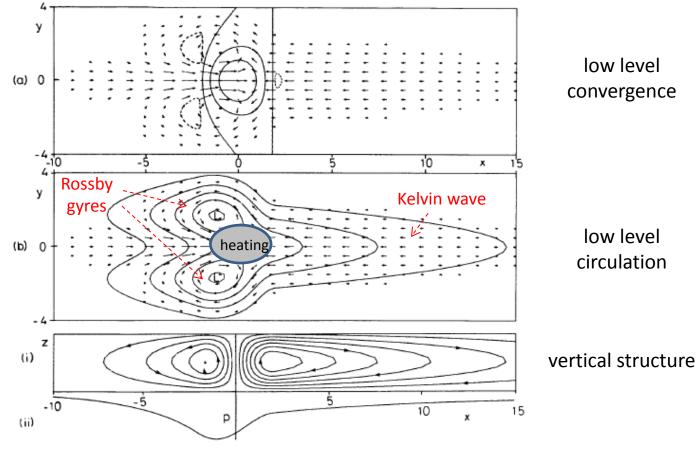




### Seasonal variations in tropical overturning circulation (Hadley cell)

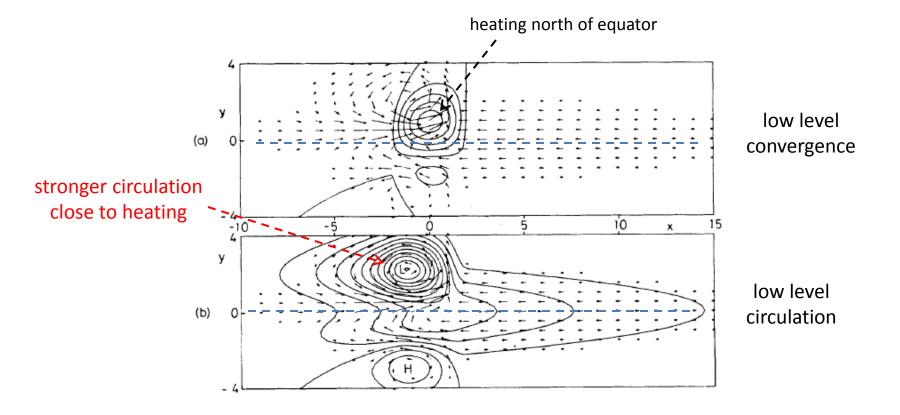


# Dynamical response to low frequency convective forcing



Gill, 1980

### Dynamical response to heating centered north of equator



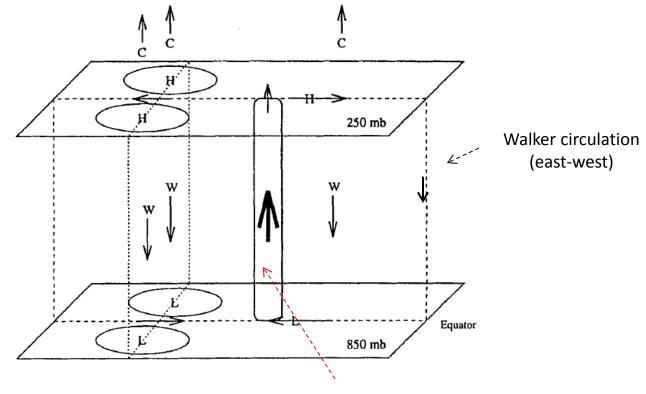
Gill, 1980

# The tropical tropopause

By E. J. HIGHWOOD\* and B. J. HOSKINS University of Reading, UK

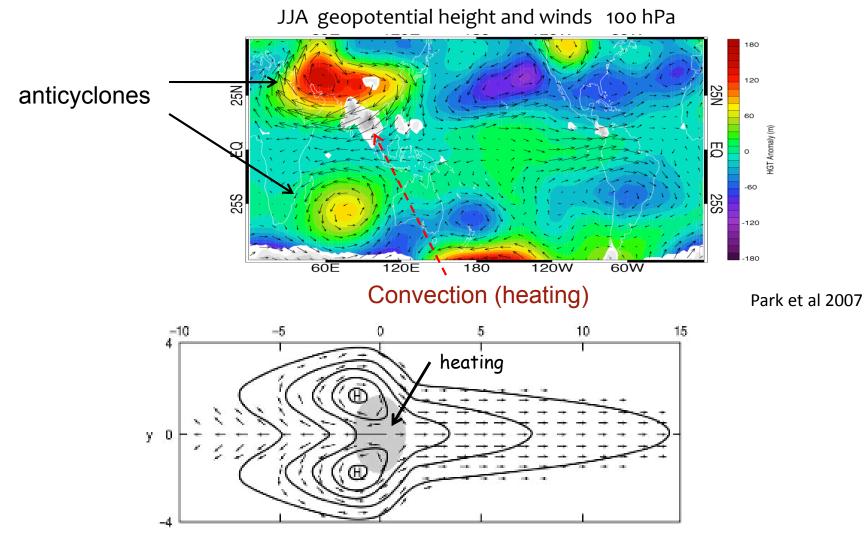
**QJRMS 1998** 

<u>vertical structure</u>: out of phase between lower troposphere and upper troposphere



deep convection / heating

# Tropical heating produces subtropical anticyclones in the UTLS



Matsuno-Gill Solution

## Key points:

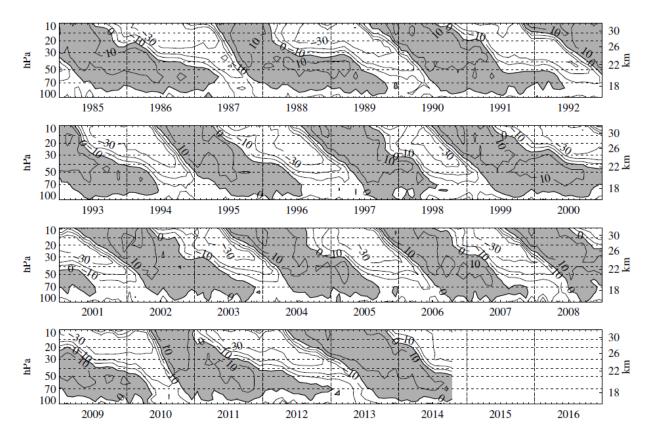
- Organized deep convection (latent heating) drives large-scale tropical circulations
- Seasonal movement between solstices (SH NH subtropics)
- Hadley and Walker overturning circulations
- Matsuno-Gill dynamical response to local heating: subtropical Rossby waves
   and equatorial Kelvin waves
- Subtropical anticyclones in UTLS (especially Asian monsoon during NH summer)

The Quasi-Biennial Oscillation (QBO) and El Nino Southern Oscillation (ENSO) are important modes of circulation in the stratosphere, troposphere and UTLS

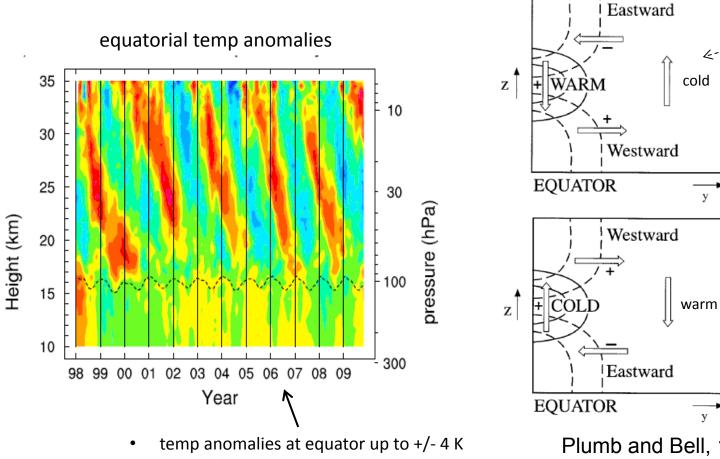
- QBO: approximate 28-month oscillation in tropical stratosphere, forced by vertically propagating waves interacting with mean flow
- ENSO: ~2-6 year time scale oscillations of convection, oceanic and atmospheric circulations. Wave effects extend into high latitudes and into tropical lower stratosphere.
- Both QBO and ENSO influence temperature, circulation and transport in UTLS

easterly descent is regular; sometimes the westerly phase 'stalls'





# Meridional circulations and temperatures linked with the QBO



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extension to middle latitudes

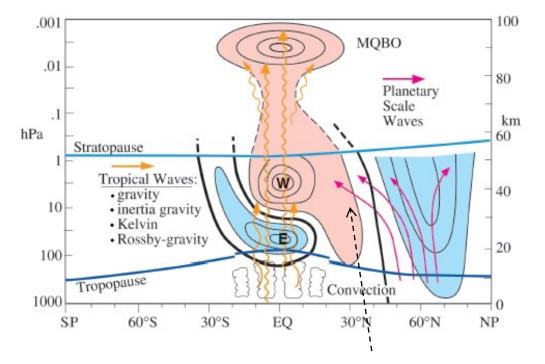
(a)

(b)

extend downward to near tropopause

Plumb and Bell, 1982b, Q.J.R. Meteorol. Soc.

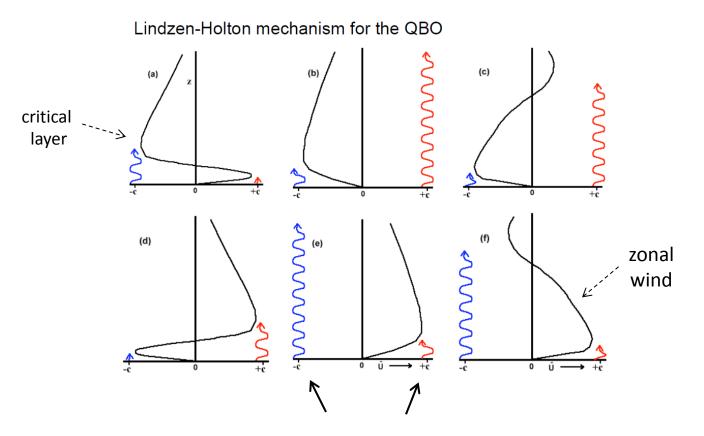
### Meridional structure and global influences of the QBO



Baldwin et al 1998

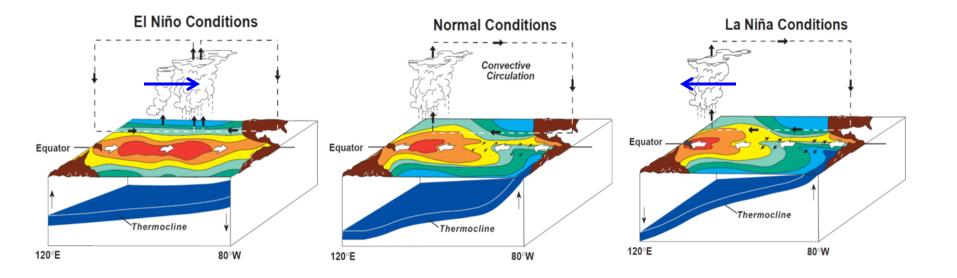
influence on high latitudes via subtropical critical lines

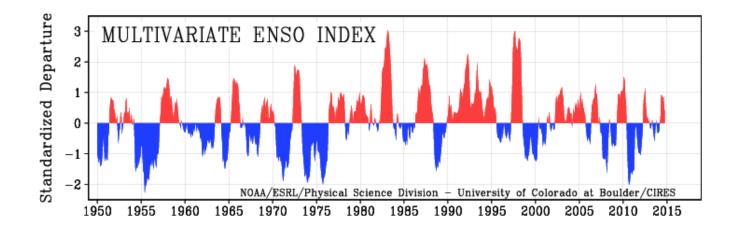
- waves propagate vertically until reaching critical level (U=c)
- wave absorption produces zonal acceleration (towards phase speed of wave)



eastward and westward propagating waves forced from troposphere

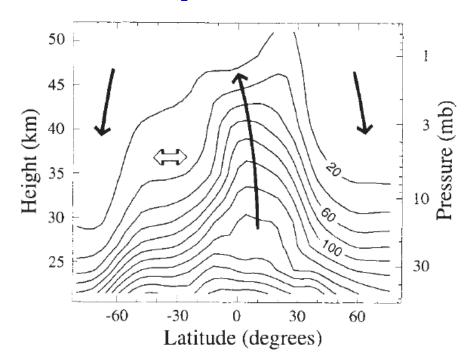
# ENSO – large-scale shifts in tropical convection, winds, temperatures





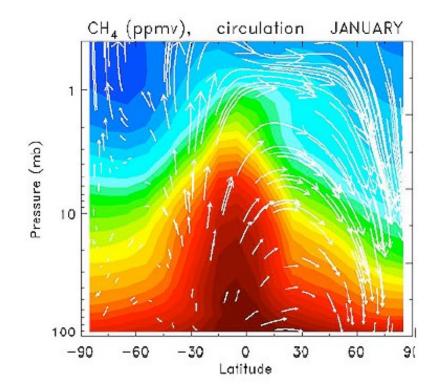
### Stratospheric tracer transport: evidence from satellite observations

- N<sub>2</sub>O is a 'tropospheric source gas'
- destroyed by photolysis (radiation) in upper stratosphere
- Source of reactive nitrogen (NO, NO2) in upper stratosphere; important for stratospheric ozone
- Behavior reflects Brewer-Dobson circulation and eddy mixing



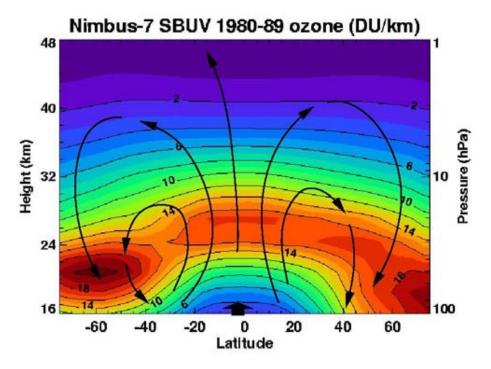
#### N<sub>2</sub>O mixing ratio

Methane  $CH_4$  is another tropospheric source gas, oxidized to  $H_2O$  in upper stratosphere

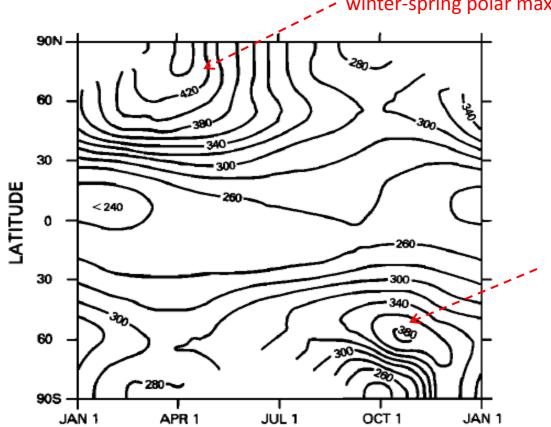


Observed ozone and Brewer-Dobson circulation

- ozone is made in the tropical stratosphere
- Short lifetime in upper stratosphere
- Long lifetime in lower stratosphere
- transport causes high latitude maximum during winter / spring



# Seasonal cycle of column ozone reflects Brewer-Dobson circulation



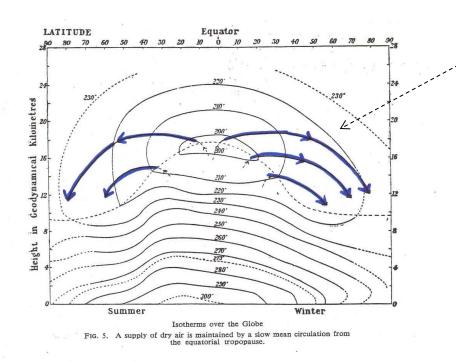
winter-spring polar maximum

Bowman and Krueger, 1982

#### EVIDENCE FOR A WORLD CIRCULATION PROVIDED BY MEASUREMENTS OF HELIUM AND WATER VAPOUR DISTRIBUTION IN THE STRATOSPHERE

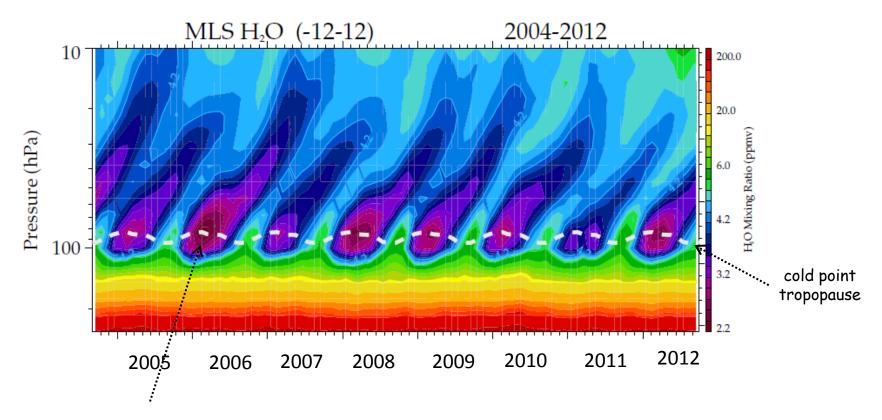
QJRMS, 1949

By A. W. BKEWER, M.Sc., A.Inst.P.



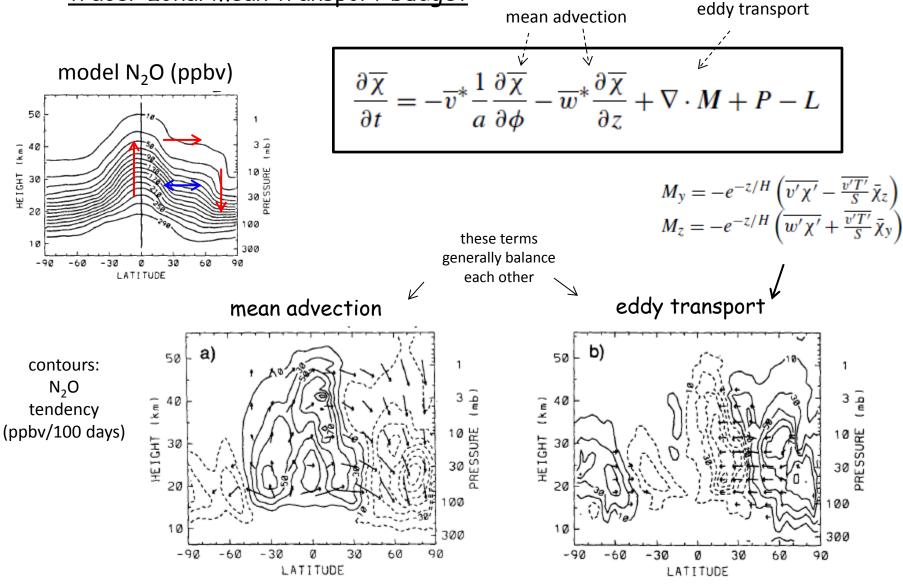
The stratosphere is extremely dry because air is dehydrated passing the cold tropical tropopause

# Tropical tape recorder observed by MLS 2004-2012



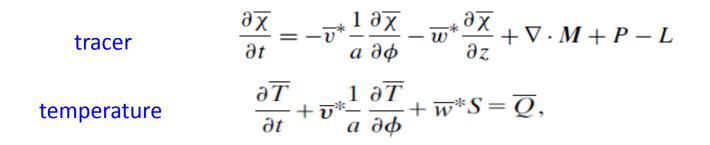
- annual cycle in tropopause temperature imparts annual cycle in H<sub>2</sub>O
- upward propagation with Brewer-Dobson circulation

# tracer zonal mean transport budget

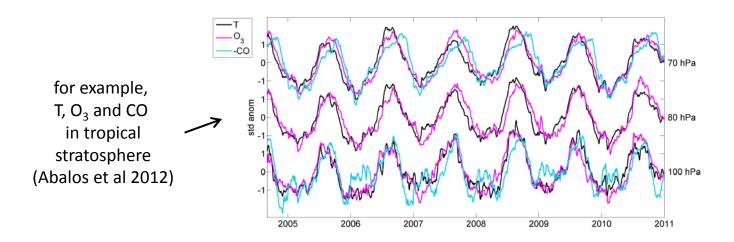


Randel et al 1994

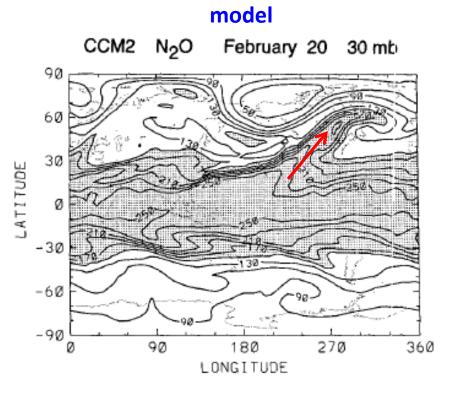
### Tracer transport equation similar to thermodynamic equation:



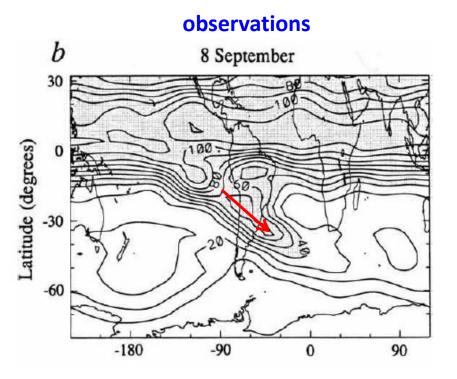
### This is why temperature and tracers are sometimes highly correlated:



### Examples of stratospheric wave mixing



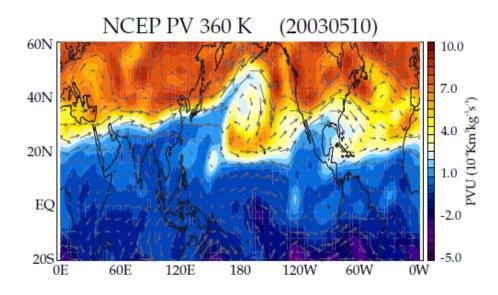
#### N<sub>2</sub>O near 35 km from CLAES instrument on UARS

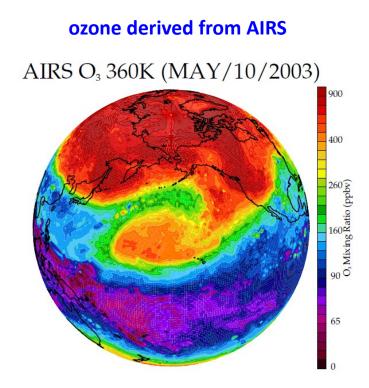


Randel et al 1993

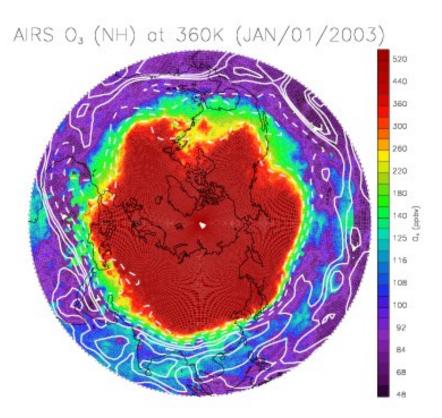
Mixing across tropopause linked to Rossby wave breaking

#### potential vorticity





# Rossby wave variability reflected in ozone near tropopause

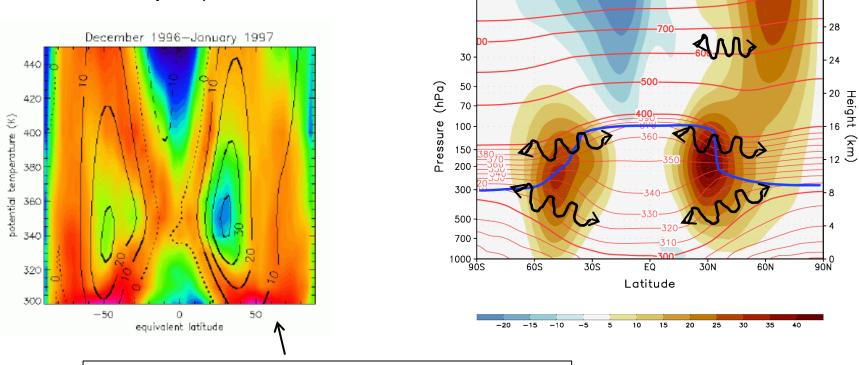


#### Effective diffusivity as a diagnostic of atmospheric transport 2. Troposphere and lower stratosphere

JGR 2000

Peter Haynes and Emily Shuckburgh

## Estimates of mixing based on stretching of PV contours in trajectory calculations



U and  $\theta$ 

10

DJF

32

important points:

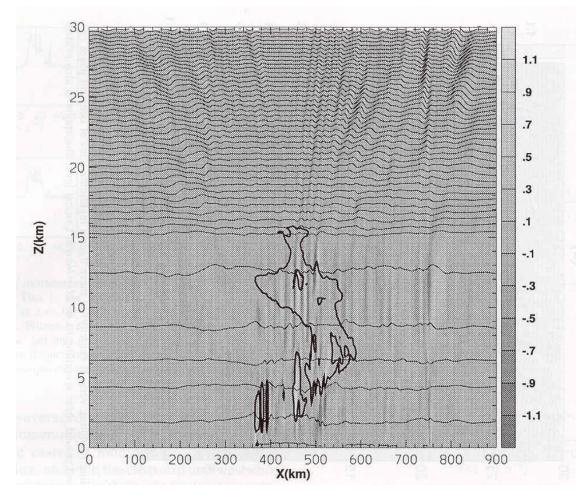
- mixing on flanks of jet (near critical lines for c ~10 m/s)
- small mixing across jet core (jet cores are mixing barriers)

#### Key points:

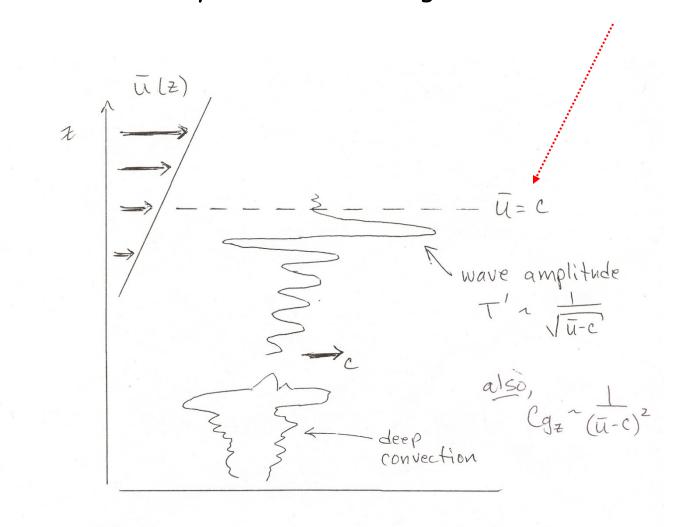
- Stratospheric transport: Brewer-Dobson circulation and wave mixing (clear behavior for tropospheric source gases)
- Stratrospheric ozone: produced in tropical stratosphere, transported to high latitudes (reflects seasonal Brewer-Dobson circulation)
- Stratospheric water vapor: dehydration near tropical cold point, strong seasonal cycle ('tape recorder')
- Tracer budgets: mean advection and eddy transports (tied to Rossby waves and critical layers)

Extra slides

# Model simulation of gravity waves forced by deep convection

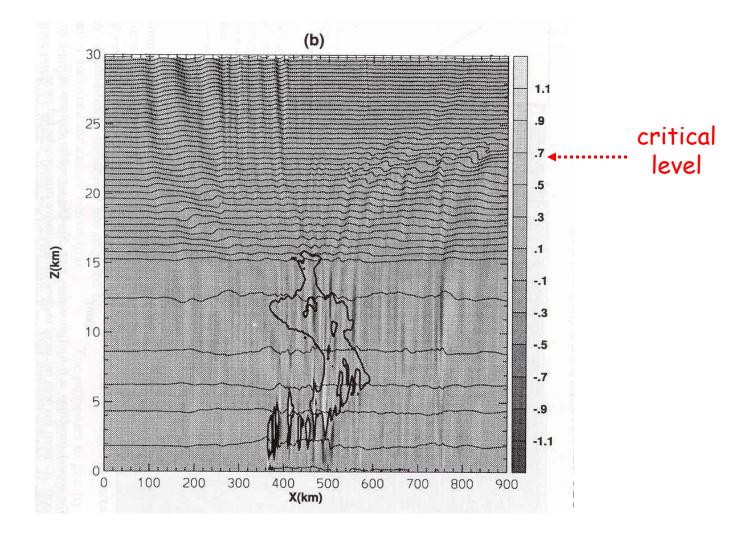


Alexander and Holton, 2000



Gravity waves interacting with a critical level

# Gravity waves interacting with a critical level



Alexander and Holton, 2000, J. Atmos. Sci.

#### Climatology of Intrusions into the Tropical Upper Troposphere

Darryn W. Waugh

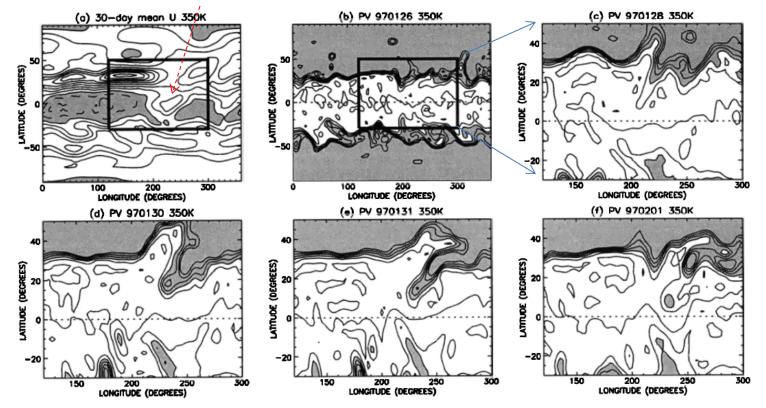
Department of Earth and Planetary Science, Johns Hopkins University, Baltimore, MD.

Lorenzo M. Polvani

Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY.

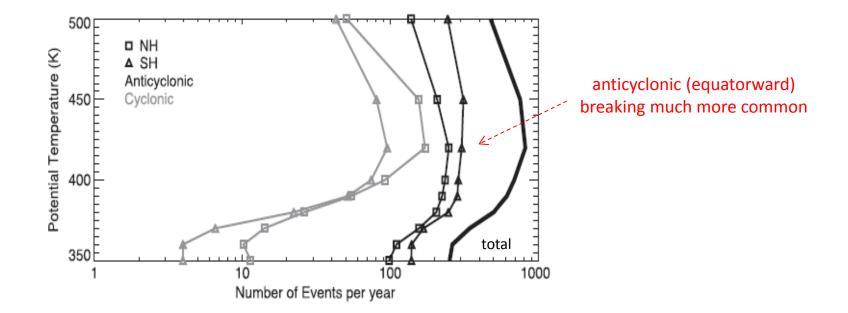
Latitudinal propagation depends on background zonal winds:

 Rossby wave propagation through westerly wind 'ducts'



#### westerly winds

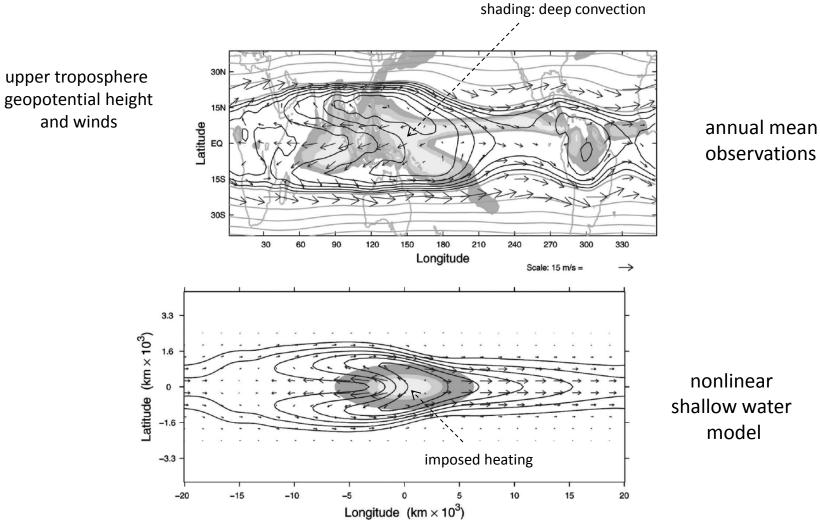
#### climatology of Rossby wave breaking events



Homeyer and Bowman 2013

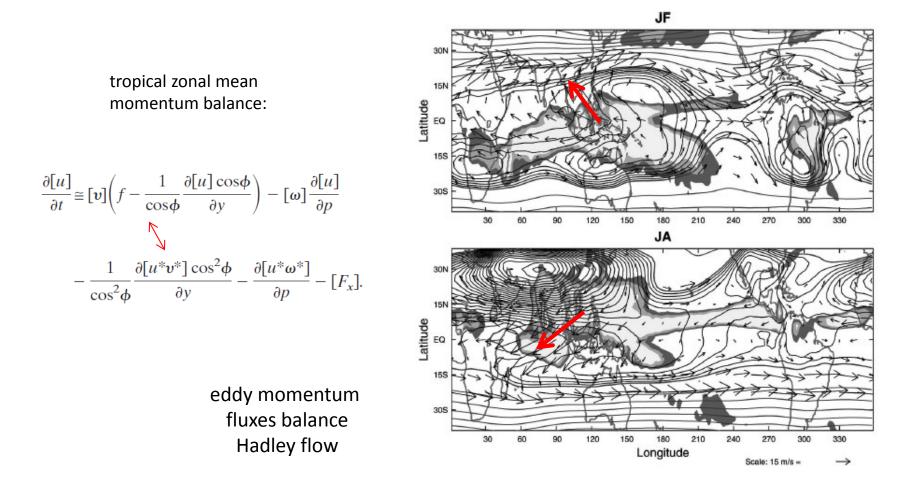
#### observations vs. model

and winds



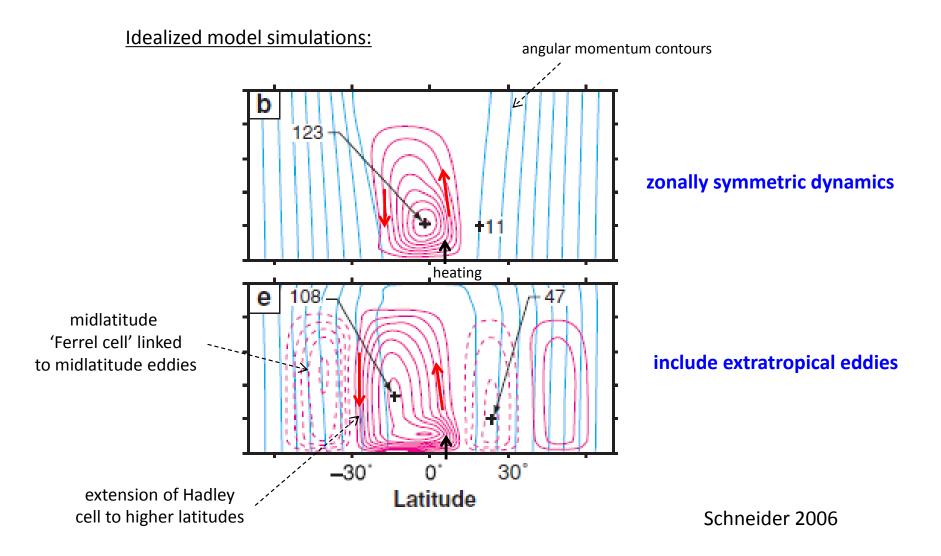
Dima et al 2005

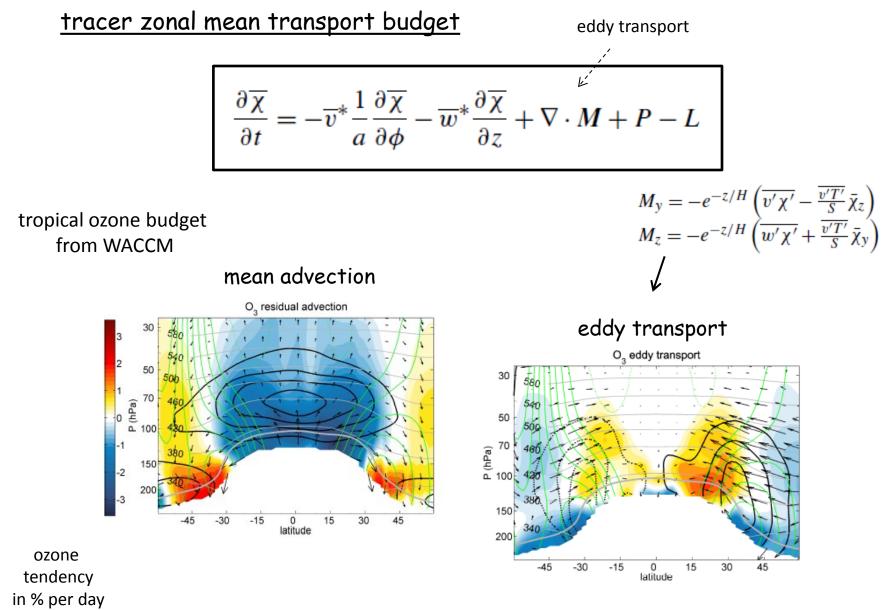
#### Seasonal variation of tropical waves



Dima et al 2005

Hadley cell interactions with extratropical eddies: (a complicated subject)





Abalos et al 2013

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