# 演習問題 3.6 解答例

## (2)

$$
\begin{aligned}
\mathrm{rot}(\mathrm{grad}\varphi) &= \nabla \times \nabla\varphi \\
&= \nabla \times \left[\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right] \\
&= \left[\frac{\partial}{\partial y}\frac{\partial\varphi}{\partial z} - \frac{\partial}{\partial z}\frac{\partial\varphi}{\partial y}, \frac{\partial}{\partial z}\frac{\partial\varphi}{\partial x} - \frac{\partial}{\partial x}\frac{\partial\varphi}{\partial z}, \frac{\partial}{\partial x}\frac{\partial\varphi}{\partial y} - \frac{\partial}{\partial y}\frac{\partial\varphi}{\partial x}\right] \\
&= [0, 0, 0] \\
&= 0.
\end{aligned}
$$

## (3)

$$
\begin{aligned}
\mathrm{div}(\mathrm{rot}\boldsymbol{A}) &= \nabla \cdot (\nabla \times \boldsymbol{A}) \\
&= \frac{\partial}{\partial x}\left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right) \\
&= \left(\frac{\partial}{\partial y}\frac{\partial}{\partial z} - \frac{\partial}{\partial z}\frac{\partial}{\partial y}\right)a_x + \left(\frac{\partial}{\partial z}\frac{\partial}{\partial x} - \frac{\partial}{\partial x}\frac{\partial}{\partial z}\right)a_y + \left(\frac{\partial}{\partial x}\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\frac{\partial}{\partial x}\right)a_z \\
&= 0.
\end{aligned}
$$

## (4)

$x$ 成分についてのみ考える

$$
\begin{aligned}
[\mathrm{rot}(\varphi\boldsymbol{A})]_x &= \frac{\partial}{\partial y}(\varphi a_z) - \frac{\partial}{\partial z}(\varphi a_y) \\
&= \frac{\partial\varphi}{\partial y}a_z + \varphi\frac{\partial a_z}{\partial y} - \left(\frac{\partial\varphi}{\partial z}a_y + \varphi\frac{\partial a_y}{\partial z}\right) \\
&= \frac{\partial\varphi}{\partial y}a_z - \frac{\partial\varphi}{\partial z}a_y + \varphi\frac{\partial a_z}{\partial y} - \varphi\frac{\partial a_y}{\partial z} \\
&= [\nabla\varphi \times \boldsymbol{A}]_x + [\varphi\nabla \times \boldsymbol{A}]_x
\end{aligned}
$$

$y, z$ 成分についても同様に考えられるので,

$$[\mathrm{rot}(\varphi \boldsymbol{A})]_x = \nabla\varphi \times \boldsymbol{A} + \varphi\nabla \times \boldsymbol{A}$$