dcpam physics の一部について

高橋芳幸

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まえがき

これは自己満足のために作っているものであり, 他人に読ませることを想定していない. したがって表現などかなりいい加減である. どこまで続くかも未知数である.

1 Basics

$$R_i = \frac{\frac{g}{\theta}\frac{\partial\theta}{\partial z}}{\left|\frac{\partial \boldsymbol{u}}{\partial z}\right|} \tag{1}$$

$$\left|\frac{\partial \boldsymbol{u}}{\partial z}\right|_{k+\frac{1}{2}} = \sqrt{\left(\frac{u_{k+1}-u_k}{z_{k+1}-z_k}\right)^2 + \left(\frac{v_{k+1}-v_k}{z_{k+1}-z_k}\right)^2} \tag{2}$$

$$R_{i,k+\frac{1}{2}} = \ldots \tag{3}$$

Is the calculation of R_i in the model correct?

The diffusion coefficients, K_m, K_h, and K_q, are evaluated based on the Mellor and Yamada level 2 scheme.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial F_{m,x}}{\partial z} \tag{4}$$

$$= g\frac{\partial F_{m,x}}{\partial p} \tag{5}$$

$$F_{m,x} = -\rho K_m \frac{\partial u}{\partial z} \tag{6}$$

...

$$\frac{u_k^{t+\Delta t} - u_k^{t-\Delta t}}{2\Delta t} = g\frac{F_{m,x,k+\frac{1}{2}}^{t+\Delta t} - F_{m,x,k-\frac{1}{2}}^{t+\Delta t}}{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}} \tag{7}$$

$$\frac{v_k^{t+\Delta t} - v_k^{t-\Delta t}}{2\Delta t} = g\frac{F_{m,y,k+\frac{1}{2}}^{t+\Delta t} - F_{m,y,k-\frac{1}{2}}^{t+\Delta t}}{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}} \tag{8}$$

$$\frac{T_k^{t+\Delta t} - T_k^{t-\Delta t}}{2\Delta t} = \frac{1}{C_p} g \frac{F_{h,k+\frac{1}{2}}^{t+\Delta t} - F_{h,k-\frac{1}{2}}^{t+\Delta t}}{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}} \tag{9}$$

$$\frac{q_k^{t+\Delta t} - q_k^{t-\Delta t}}{2\Delta t} = g \frac{F_{q,k+\frac{1}{2}}^{t+\Delta t} - F_{q,k-\frac{1}{2}}^{t+\Delta t}}{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}} \tag{10}$$

$$F_{m,x,k+\frac{1}{2}} = -(TC)_{m,k+\frac{1}{2}} (u_{k+1} - u_k) \tag{11}$$

$$= -(TC)_{m,k+\frac{1}{2}} u_{k+1} + (TC)_{m,k+\frac{1}{2}} u_k \tag{12}$$

$$F_{m,y,k+\frac{1}{2}} = -(TC)_{m,k+\frac{1}{2}} (v_{k+1} - v_k) \tag{13}$$

$$= -(TC)_{m,k+\frac{1}{2}} v_{k+1} + (TC)_{m,k+\frac{1}{2}} v_k \tag{14}$$

$$F_{h,k+\frac{1}{2}} = -C_p P_{k+\frac{1}{2}} (TC)_{h,k+\frac{1}{2}} \left(\frac{T_{k+1}}{P_{k+1}} - \frac{T_k}{P_k} \right) \tag{15}$$

$$= -C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} T_{k+1} + C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}} T_k \tag{16}$$

$$F_{q,k+\frac{1}{2}} = -(TC)_{q,k+\frac{1}{2}} (q_{k+1} - q_k) \tag{17}$$

$$= -(TC)_{q,k+\frac{1}{2}} q_{k+1} + (TC)_{q,k+\frac{1}{2}} q_k \tag{18}$$

$$(TC)_{m,k+\frac{1}{2}} = \rho_{k+\frac{1}{2}} K_{m,k+\frac{1}{2}} \frac{1}{z_{k+1} - z_k} \tag{19}$$

$$(TC)_{h,k+\frac{1}{2}} = \rho_{k+\frac{1}{2}} K_{h,k+\frac{1}{2}} \frac{1}{z_{k+1} - z_k} \tag{20}$$

$$(TC)_{q,k+\frac{1}{2}} = \rho_{k+\frac{1}{2}} K_{q,k+\frac{1}{2}} \frac{1}{z_{k+1} - z_k} \tag{21}$$

$$\rho_{k+\frac{1}{2}} = p_{k+\frac{1}{2}} / (R T_{k+\frac{1}{2}}) \tag{22}$$

for $k = 1$

$$F_{m,x,k-\frac{1}{2}} = -(TC)_{m,k-\frac{1}{2}} u_1 \tag{23}$$

$$F_{m,y,k-\frac{1}{2}} = -(TC)_{m,k-\frac{1}{2}} v_1 \tag{24}$$

$$F_{h,k-\frac{1}{2}} = -C_p P_{k-\frac{1}{2}} (TC)_{h,k-\frac{1}{2}} \left(\frac{T_k}{P_k} - \frac{T_s}{P_{k-\frac{1}{2}}} \right) \tag{25}$$

$$= -C_p (TC)_{h,k-\frac{1}{2}} \frac{P_{k-\frac{1}{2}}}{P_k} T_k + C_p (TC)_{h,k-\frac{1}{2}} T_s \tag{26}$$

$$F_{q,k-\frac{1}{2}} = -\epsilon (TC)_{q,k-\frac{1}{2}} (q_k - q_s^*) \tag{27}$$

$$= -\epsilon (TC)_{q,k-\frac{1}{2}} q_k + \epsilon (TC)_{q,k-\frac{1}{2}} q_s^* \tag{28}$$

前のコメント:
P (Exner function) の定義をどうするかを考えなければ. それによっては T_s を $P_{\frac{1}{2}}$ で割って, $F_{h,\frac{1}{2}}$ 全体に $P_{\frac{1}{2}}$ をかけないといけないんじゃないだろうか? 今は, $P = \left(\frac{p}{p_{\frac{1}{2}}}\right)^{\kappa}$ と定義しているようなので, T_s はそのままで良いことになっている.
P_1 をどうするかは別途.

... しかし, 熱フラックスを「温度の輸送フラックス」で評価していいのだろうか? 温位の輸送フラックスじゃなくて?

今のコメント (2008/01/06):

上の式では地面フラックスに $P_{\frac{1}{2}} = P_{k-\frac{1}{2}}$ をかけることにしてある. この式では P (Exner function) の定義によらない.

ただし, コード修正時には Richardson 数の計算も直すこと. 今は「平均場の温位」を定数として使っている. (2008/01/25)

$$
\begin{aligned}
(TC)_{m,k-\frac{1}{2}} &= \rho_s C_d \left| \boldsymbol{u}_k \right| && (29)\\
(TC)_{h,k-\frac{1}{2}} &= \rho_s C_h \left| \boldsymbol{u}_k \right| && (30)\\
(TC)_{q,k-\frac{1}{2}} &= \rho_s C_q \left| \boldsymbol{u}_k \right| && (31)\\
\rho_s &= \frac{p_s}{RT_0} && (32)
\end{aligned}
$$

最後は T_0 (大気の温度) なのかね? T_s ではなくて?

for $k = k_{max}$

$$
\begin{aligned}
F_{m,x,k_{max}+\frac{1}{2}} &= 0 && (33)\\
F_{m,y,k_{max}+\frac{1}{2}} &= 0 && (34)\\
F_{h,k_{max}+\frac{1}{2}} &= 0 && (35)\\
F_{q,kamx+\frac{1}{2}} &= 0 && (36)
\end{aligned}
$$

2 Difference equation for diffusion of momentum

$$
\begin{aligned}
&\frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} \left(u_k^{t+\Delta t} - u_k^{t-\Delta t} \right) && (37)\\
&= F^{t+\Delta t}_{m,x,k+\frac{1}{2}} - F^{t+\Delta t}_{m,x,k-\frac{1}{2}} && (38)\\
&= F^{t+\Delta t}_{m,x,k+\frac{1}{2}} - F^{t+\Delta t}_{m,x,k-\frac{1}{2}}\\
&\quad -F^{t-\Delta t}_{m,x,k+\frac{1}{2}} + F^{t-\Delta t}_{m,x,k-\frac{1}{2}}\\
&\quad +F^{t-\Delta t}_{m,x,k+\frac{1}{2}} - F^{t-\Delta t}_{m,x,k-\frac{1}{2}} && (39)
\end{aligned}
$$

for $2 \le k \le k_{max} - 1$

$$
\begin{aligned}
&\frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} \left(u_k^{t+\Delta t} - u_k^{t-\Delta t} \right) && (40)\\
&= -(TC)_{m,k+\frac{1}{2}} u_{k+1}^{t+\Delta t} + (TC)_{m,k+\frac{1}{2}} u_k^{t+\Delta t}\\
&\qquad +(TC)_{m,k-\frac{1}{2}} u_k^{t+\Delta t} - (TC)_{m,k-\frac{1}{2}} u_{k-1}^{t+\Delta t}\\
&\quad +(TC)_{m,k+\frac{1}{2}} u_{k+1}^{t-\Delta t} - (TC)_{m,k+\frac{1}{2}} u_k^{t-\Delta t}\\
&\qquad -(TC)_{m,k-\frac{1}{2}} u_k^{t-\Delta t} + (TC)_{m,k-\frac{1}{2}} u_{k-1}^{t-\Delta t}\\
&\quad +F^{t-\Delta t}_{m,x,k+\frac{1}{2}} - F^{t-\Delta t}_{m,x,k-\frac{1}{2}} && (41)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -(TC)_{m,k-\frac{1}{2}}\left(u_{k-1}^{t+\Delta t}-u_{k-1}^{t-\Delta t}\right) \\
& +\left((TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}}\right)\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \\
& -(TC)_{m,k+\frac{1}{2}}\left(u_{k+1}^{t+\Delta t}-u_{k+1}^{t-\Delta t}\right) \\
& +F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t} \qquad (42)
\end{aligned}
$$

より

$$
\begin{aligned}
& -(TC)_{m,k-\frac{1}{2}}\left(u_{k-1}^{t+\Delta t}-u_{k-1}^{t-\Delta t}\right) \\
& \quad +\left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}}\right)\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \\
& \quad -(TC)_{m,k+\frac{1}{2}}\left(u_{k+1}^{t+\Delta t}-u_{k+1}^{t-\Delta t}\right) \qquad (43) \\
= \quad & -\left(F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t}\right) \qquad (44)
\end{aligned}
$$

for $k = 1$

$$
\begin{aligned}
& \frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \qquad (45) \\
= \quad & -(TC)_{m,k+\frac{1}{2}}u_{k+1}^{t+\Delta t}+(TC)_{m,k+\frac{1}{2}}u_{k}^{t+\Delta t} \qquad (46) \\
& \quad +(TC)_{m,k-\frac{1}{2}}u_{k}^{t+\Delta t} \qquad (47) \\
& +(TC)_{m,k+\frac{1}{2}}u_{k+1}^{t-\Delta t}-(TC)_{m,k+\frac{1}{2}}u_{k}^{t-\Delta t} \qquad (48) \\
& \quad -(TC)_{m,k-\frac{1}{2}}u_{k}^{t-\Delta t} \qquad (49) \\
& +F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t} \qquad (50) \\
= \quad & \left((TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}}\right)\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \qquad (51) \\
& -(TC)_{m,k+\frac{1}{2}}\left(u_{k+1}^{t+\Delta t}-u_{k+1}^{t-\Delta t}\right) \qquad (52) \\
& +F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t} \qquad (53)
\end{aligned}
$$

より

$$
\begin{aligned}
& \left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}}\right)\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \qquad (54) \\
& \quad -(TC)_{m,k+\frac{1}{2}}\left(u_{k+1}^{t+\Delta t}-u_{k+1}^{t-\Delta t}\right) \qquad (55) \\
= \quad & -\left(F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t}\right) \qquad (56)
\end{aligned}
$$

for $k = k_{max}$

$$
\begin{aligned}
& \frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(u_{k}^{t+\Delta t}-u_{k}^{t-\Delta t}\right) \qquad (57) \\
= \quad & 0 \qquad (58) \\
& \quad +(TC)_{m,k-\frac{1}{2}}u_{k}^{t+\Delta t}-(TC)_{m,k-\frac{1}{2}}u_{k-1}^{t+\Delta t} \qquad (59) \\
& 0 \qquad (60) \\
& \quad -(TC)_{m,k-\frac{1}{2}}u_{k}^{t-\Delta t}+(TC)_{m,k-\frac{1}{2}}u_{k-1}^{t-\Delta t} \qquad (61) \\
& +F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t} \qquad (62) \\
= \quad & -(TC)_{m,k-\frac{1}{2}}\left(u_{k-1}^{t+\Delta t}-u_{k-1}^{t-\Delta t}\right) \qquad (63)
\end{aligned}
$$

$$+(TC)_{m,k-\frac{1}{2}}\left(u_k^{t+\Delta t}-u_k^{t-\Delta t}\right) \tag{64}$$
$$+F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t} \tag{65}$$

より

$$-(TC)_{m,k-\frac{1}{2}}\left(u_{k-1}^{t+\Delta t}-u_{k-1}^{t-\Delta t}\right) \tag{66}$$
$$+\left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}}\right)\left(u_k^{t+\Delta t}-u_k^{t-\Delta t}\right) \tag{67}$$
$$= -\left(F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t}\right) \tag{68}$$

$$\boldsymbol{A}\boldsymbol{x}_u=\boldsymbol{G}_u \tag{69}$$

$$\boldsymbol{x}_u = \left(u_1^{t+\Delta t}-u_1^{t-\Delta t},u_2^{t+\Delta t}-u_2^{t-\Delta t},...,u_{k_{max}}^{t+\Delta t}-u_{k_{max}}^{t-\Delta t}\right), \tag{70}$$
$$\boldsymbol{G}_u = \left(g_{u,1},g_{u,2},...,g_{u,k_{max}}\right), \tag{71}$$
$$g_{u,k} = -\left(F_{m,x,k+\frac{1}{2}}^{t-\Delta t}-F_{m,x,k-\frac{1}{2}}^{t-\Delta t}\right) \tag{72}$$

for $2\le k\le k_{max}-1$

$$a_{k,k-1} = -(TC)_{m,k-\frac{1}{2}} \tag{73}$$
$$a_{k,k} = -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}} \tag{74}$$
$$a_{k,k+1} = -(TC)_{m,k+\frac{1}{2}} \tag{75}$$

for $k=1$

$$a_{k,k} = -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}}+(TC)_{m,k+\frac{1}{2}} \tag{76}$$
$$a_{k,k+1} = -(TC)_{m,k+\frac{1}{2}} \tag{77}$$

for $k=k_{max}$

$$a_{k,k-1} = -(TC)_{m,k-\frac{1}{2}} \tag{78}$$
$$a_{k,k} = -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{m,k-\frac{1}{2}} \tag{79}$$

南北風に関しては, 東西風と同様に以下のように書くことができる.

$$\boldsymbol{A}\boldsymbol{x}_v=\boldsymbol{G}_v \tag{80}$$

ここで, ...

$$\boldsymbol{x}_v = \left(v_1^{t+\Delta t}-v_1^{t-\Delta t},v_2^{t+\Delta t}-v_2^{t-\Delta t},...,v_{k_{max}}^{t+\Delta t}-v_{k_{max}}^{t-\Delta t}\right), \tag{81}$$
$$\boldsymbol{G}_v = \left(g_{v,1},g_{v,2},...,g_{v,k_{max}}\right), \tag{82}$$
$$g_{v,k} = -\left(F_{m,y,k+\frac{1}{2}}^{t-\Delta t}-F_{m,y,k-\frac{1}{2}}^{t-\Delta t}\right) \tag{83}$$

3 Difference equation for diffusion of heat

$$C_p \frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \tag{84}$$

$$= F_{h,k+\frac{1}{2}}^{t+\Delta t} - F_{h,k-\frac{1}{2}}^{t+\Delta t} \tag{85}$$

$$= F_{h,k+\frac{1}{2}}^{t+\Delta t} - F_{h,k-\frac{1}{2}}^{t+\Delta t} \tag{86}$$

$$-F_{h,k+\frac{1}{2}}^{t-\Delta t} + F_{h,k-\frac{1}{2}}^{t-\Delta t} \tag{87}$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \tag{88}$$

for $2 \leq k \leq k_{max} - 1$

$$C_p \frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \tag{89}$$

$$= -C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} T_{k+1}^{t+\Delta t} + C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}} T_k^{t+\Delta t} \tag{90}$$

$$+C_p \frac{P_{k-\frac{1}{2}}}{P_k} (TC)_{h,k-\frac{1}{2}} T_k^{t+\Delta t} - C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}} (TC)_{h,k-\frac{1}{2}} T_{k-1}^{t+\Delta t} \tag{91}$$

$$+C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} T_{k+1}^{t-\Delta t} - C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}} T_k^{t-\Delta t} \tag{92}$$

$$-C_p \frac{P_{k-\frac{1}{2}}}{P_k} (TC)_{h,k-\frac{1}{2}} T_k^{t-\Delta t} + C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}} (TC)_{h,k-\frac{1}{2}} T_{k-1}^{t-\Delta t} \tag{93}$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \tag{94}$$

$$= -C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}} (TC)_{h,k-\frac{1}{2}} \left(T_{k-1}^{t+\Delta t} - T_{k-1}^{t-\Delta t}\right) \tag{95}$$

$$+\left(C_p \frac{P_{k-\frac{1}{2}}}{P_k} (TC)_{h,k-\frac{1}{2}} + C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}}\right) \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \tag{96}$$

$$-C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} \left(T_{k+1}^{t+\Delta t} - T_{k+1}^{t-\Delta t}\right) \tag{97}$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \tag{98}$$

より

$$-C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}} (TC)_{h,k-\frac{1}{2}} \left(T_{k-1}^{t+\Delta t} - T_{k-1}^{t-\Delta t}\right) \tag{99}$$

$$+\left(-C_p \frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} + C_p \frac{P_{k-\frac{1}{2}}}{P_k} (TC)_{h,k-\frac{1}{2}} + C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}}\right) \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \tag{100}$$

$$-C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} \left(T_{k+1}^{t+\Delta t} - T_{k+1}^{t-\Delta t}\right) \tag{101}$$

$$= -\left(F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t}\right) \tag{102}$$

for $k = 1$

$$C_p \frac{1}{2\Delta t} \frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \tag{103}$$

$$= -C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}} (TC)_{h,k+\frac{1}{2}} T_{k+1}^{t+\Delta t} + C_p \frac{P_{k+\frac{1}{2}}}{P_k} (TC)_{h,k+\frac{1}{2}} T_k^{t+\Delta t} \tag{104}$$

$$+C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} T_k^{t+\Delta t} - C_p (TC)_{h,k-\frac{1}{2}} T_s^{t+\Delta t} \quad (105)$$

$$+C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}}(TC)_{h,k+\frac{1}{2}} T_{k+1}^{t-\Delta t} - C_p \frac{P_{k+\frac{1}{2}}}{P_k}(TC)_{h,k+\frac{1}{2}} T_k^{t-\Delta t} \quad (106)$$

$$-C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} T_k^{t-\Delta t} + C_p (TC)_{h,k-\frac{1}{2}} T_s^{t-\Delta t} \quad (107)$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \quad (108)$$

$$= -C_p (TC)_{h,\frac{1}{2}} \left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \quad (109)$$

$$+\left(C_p \frac{P_{k+\frac{1}{2}}}{P_k}(TC)_{h,k+\frac{1}{2}} + C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}}\right)\left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \quad (110)$$

$$-C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}}(TC)_{h,k+\frac{1}{2}} \left(T_{k+1}^{t+\Delta t} - T_{k+1}^{t-\Delta t}\right) \quad (111)$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \quad (112)$$

より

$$-C_p (TC)_{h,k-\frac{1}{2}} \left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \quad (113)$$

$$+\left(-C_p \frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} + C_p \frac{P_{k+\frac{1}{2}}}{P_k}(TC)_{h,k+\frac{1}{2}} + C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}}\right)\left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \quad (114)$$

$$-C_p \frac{P_{k+\frac{1}{2}}}{P_{k+1}}(TC)_{h,k+\frac{1}{2}} \left(T_{k+1}^{t+\Delta t} - T_{k+1}^{t-\Delta t}\right) \quad (115)$$

$$= -\left(F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t}\right) \quad (116)$$

地面フラックスに $P_{\frac{1}{2}} = P_{k-\frac{1}{2}}$ をかけることにしたので，ここでも $P_{k-\frac{1}{2}}$ がかかっている．
ただし，コード修正時には Richardson 数の計算も直すこと．今は「平均場の温位」を定数として使っている．(2008/01/25)

for $k = k_{max}$

$$C_p \frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g}\left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \quad (117)$$

$$= 0 \quad (118)$$

$$+C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} T_k^{t+\Delta t} - C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} T_{k-1}^{t+\Delta t} \quad (119)$$

$$+0 \quad (120)$$

$$-C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} T_k^{t-\Delta t} + C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} T_{k-1}^{t-\Delta t} \quad (121)$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \quad (122)$$

$$= -C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} \left(T_{k-1}^{t+\Delta t} - T_{k-1}^{t-\Delta t}\right) \quad (123)$$

$$+C_p \frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} \left(T_k^{t+\Delta t} - T_k^{t-\Delta t}\right) \quad (124)$$

$$+F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t} \quad (125)$$

より

$$-C_p \frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} \left(T_{k-1}^{t+\Delta t} - T_{k-1}^{t-\Delta t}\right) \quad (126)$$

$$+\left(-C_p\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+C_p\frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}}\right)\left(T_k^{t+\Delta t}-T_k^{t-\Delta t}\right) \quad (127)$$

$$= \ -\left(F_{h,k+\frac{1}{2}}^{t-\Delta t}-F_{h,k-\frac{1}{2}}^{t-\Delta t}\right) \quad (128)$$

$$\boldsymbol{B}\boldsymbol{x}_h = \boldsymbol{G}_h \quad (129)$$

$$\boldsymbol{x}_h = \left(T_1^{t+\Delta t}-T_1^{t-\Delta t}, T_2^{t+\Delta t}-T_2^{t-\Delta t}, ..., T_{k_{max}}^{t+\Delta t}-T_{k_{max}}^{t-\Delta t}\right), \quad (130)$$

$$\boldsymbol{G}_h = \left(g_{h,1}, g_{h,2}, ..., g_{h,k_{max}}\right), \quad (131)$$

$$g_{h,k} = -\left(F_{h,k+\frac{1}{2}}^{t-\Delta t}-F_{h,k-\frac{1}{2}}^{t-\Delta t}\right) \quad (132)$$

本当はこうは書けないよね. T_s がいるから.

for $2 \le k \le k_{max}-1$

$$b_{k,k-1} = -C_p\frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} \quad (133)$$

$$b_{k,k} = -C_p\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+C_p\frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}}+C_p\frac{P_{k+\frac{1}{2}}}{P_k}(TC)_{h,k+\frac{1}{2}} \quad (134)$$

$$b_{k,k+1} = -C_p\frac{P_{k+\frac{1}{2}}}{P_{k+1}}(TC)_{h,k+\frac{1}{2}} \quad (135)$$

for $k=1$

$$b_{k,k-1} = -C_p(TC)_{h,k-\frac{1}{2}} \quad (136)$$

$$b_{k,k} = -C_p\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+C_p\frac{P_{k+\frac{1}{2}}}{P_k}(TC)_{h,k+\frac{1}{2}}+C_p\frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} \quad (137)$$

$$b_{k,k+1} = -C_p\frac{P_{k+\frac{1}{2}}}{P_{k+1}}(TC)_{h,k+\frac{1}{2}} \quad (138)$$

地面フラックスに $P_{\frac{1}{2}}=P_{k-\frac{1}{2}}$ をかけることにしたので, ここでも $P_{k-\frac{1}{2}}$ がかかっている. (上にも同じこと書いてあるけど.)

ただし, コード修正時には Richardson 数の計算も直すこと. 今は「平均場の温位」を定数として使っている. (2008/01/25)

for $k=k_{max}$

$$b_{k,k-1} = -C_p\frac{P_{k-\frac{1}{2}}}{P_{k-1}}(TC)_{h,k-\frac{1}{2}} \quad (139)$$

$$b_{k,k} = -C_p\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+C_p\frac{P_{k-\frac{1}{2}}}{P_k}(TC)_{h,k-\frac{1}{2}} \quad (140)$$

4 Surface energy balance

Energy balance at the surface is as follows:

$$C_s\frac{\partial T_s}{\partial t} = -F_{SR}-F_{LR}-F_{h,\frac{1}{2}}-LF_{q,\frac{1}{2}}+F_g. \quad (141)$$

$$C_s \frac{T_s^{t+\Delta t} - T_s^{t-\Delta t}}{2\Delta t} = -F_{SR} - F_{LR} - F_{h,\frac{1}{2}} - LF_{q,\frac{1}{2}} + F_g. \tag{142}$$

where C_s is the surface heat capacity.

Energy balance for surface with infinite heat capacity is as follows:

$$\frac{\partial T_s}{\partial t} = 0. \tag{143}$$

$$T_s^{t+\Delta t} - T_s^{t-\Delta t} = 0. \tag{144}$$

This corresponds to the condition of fixed surface temperature.

For the case with finite surface heat capacity,

$$\frac{C_s}{2\Delta t}\left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \tag{145}$$

$$= -F_{SR}^{t+\Delta t} - F_{LR}^{t+\Delta t} - F_{h,\frac{1}{2}}^{t+\Delta t} + F_{h,\frac{1}{2}}^{t-\Delta t} - F_{h,\frac{1}{2}}^{t-\Delta t} - LF_{q,\frac{1}{2}}^{t+\Delta t} + LF_{q,\frac{1}{2}}^{t-\Delta t} - LF_{q,\frac{1}{2}}^{t-\Delta t} + F_g^{t+\Delta t} \tag{146}$$

$$= -F_{SR}^{t+\Delta t} - F_{LR}^{t+\Delta t} \tag{147}$$

$$+C_p \frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}} T_1^{t+\Delta t} - C_p (TC)_{h,\frac{1}{2}} T_s^{t+\Delta t} \tag{148}$$

$$-C_p \frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}} T_1^{t-\Delta t} + C_p (TC)_{h,\frac{1}{2}} T_s^{t-\Delta t} - F_{h,\frac{1}{2}}^{t-\Delta t} \tag{149}$$

$$+L\epsilon (TC)_{q,\frac{1}{2}} q_1^{t+\Delta t} - L\epsilon (TC)_{q,\frac{1}{2}} q_s^{*,t+\Delta t} \tag{150}$$

$$-L\epsilon (TC)_{q,\frac{1}{2}} q_1^{t-\Delta t} + L\epsilon (TC)_{q,\frac{1}{2}} q_s^{*,t-\Delta t} - LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{151}$$

$$+F_g^{t+\Delta t} \tag{152}$$

$$= -F_{SR}^{t+\Delta t} \tag{153}$$

$$-F_{LR}\left(T_s^{t-\Delta t}, T_1^{t-\Delta t}\right) - \frac{\partial F_{LR}}{\partial T_s}\left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) - \frac{\partial F_{LR}}{\partial T_1}\left(T_1^{t+\Delta t} - T_1^{t-\Delta t}\right) \tag{154}$$

$$+C_p \frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}\left(T_1^{t+\Delta t} - T_1^{t-\Delta t}\right) - C_p (TC)_{h,\frac{1}{2}}\left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \tag{155}$$

$$-F_{h,\frac{1}{2}}^{t-\Delta t} \tag{156}$$

$$+L\epsilon (TC)_{q,\frac{1}{2}}\left(q_1^{t+\Delta t} - q_1^{t-\Delta t}\right) - L\epsilon (TC)_{q,\frac{1}{2}} \frac{\partial q_s^*}{\partial T}\left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \tag{157}$$

$$-LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{158}$$

$$+F_g^{t+\Delta t} \tag{159}$$

$$= -F_{SR}^{t+\Delta t} - F_{LR}\left(T_s^{t-\Delta t}, T_1^{t-\Delta t}\right) - F_{h,\frac{1}{2}}^{t-\Delta t} - LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{160}$$

$$+\left(-C_p (TC)_{h,\frac{1}{2}} - \frac{\partial F_{LR}}{\partial T_s} - L\epsilon (TC)_{q,\frac{1}{2}} \frac{\partial q_s^*}{\partial T}\right)\left(T_s^{t+\Delta t} - T_s^{t-\Delta t}\right) \tag{161}$$

$$+\left(C_p \frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}} - \frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t} - T_1^{t-\Delta t}\right) \tag{162}$$

$$+L\epsilon (TC)_{q,\frac{1}{2}}\left(q_1^{t+\Delta t} - q_1^{t-\Delta t}\right) \tag{163}$$

$$+F_g^{t+\Delta t} \tag{164}$$

より

$$-L\epsilon (TC)_{q,\frac{1}{2}}\left(q_1^{t+\Delta t} - q_1^{t-\Delta t}\right) \tag{165}$$

$$+\left(\frac{C_s}{2\Delta t}+C_p(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_s}+L\epsilon(TC)_{q,\frac{1}{2}}\frac{\partial q_s^*}{\partial T}\right)\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \quad (166)$$

$$+\left(-C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) \quad (167)$$

$$= -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t} \quad (168)$$

$$+F_g^{t+\Delta t} \quad (169)$$

for the case with finite heat capacity ($k=0$),

$$b_{k,k-1} = -L\epsilon(TC)_{q,\frac{1}{2}} \quad (170)$$

$$b_{k,k} = \frac{C_s}{2\Delta t}+C_p(TC)_{h,\frac{1}{2}}+L\epsilon(TC)_{q,\frac{1}{2}}\frac{\partial q_s^*}{\partial T}+\frac{\partial F_{LR}}{\partial T_s} \quad (171)$$

$$b_{k,k+1} = -C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_1} \quad (172)$$

$$g_{h,k} = -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_g^{t+\Delta t} \quad (173)$$

バケツモデルを使う時には (表現が不正確だが), $b_{k,k-1}=0$ とし, $b_{k,k}$ の $L\epsilon(TC)_{q,\frac{1}{2}}\frac{\partial q_s^*}{\partial T}$ を削除.
$b_{k,k-1}$ にかかるのは L ではないか? プログラムは C_p がかかっているが
修正点:

- xyaa_SurfQVapMtx(:,:,0,1) in the subroutine SurfaceFlux.

for the case with infinite heat capacity ($k=0$),

$$b_{k,k-1} = 0 \quad (174)$$

$$b_{k,k} = 1 \quad (175)$$

$$b_{k,k+1} = 0 \quad (176)$$

$$g_{h,k} = 0. \quad (177)$$

これ, 海のときのエネルギーバランスはおかしいのでは?
修正点:

- xyz_DQVapDt in the subroutine PhyImplTendency,
- xyza_TempQVapLUMtx(i,j,0,-1), xyza_TempQVapLUMtx(i,j,0,0), xyza_TempQVapLUMtx(i,j,0, 1) in the subroutine PhyImplTendency,
- xyz_DelTempQVap(i,j,0) in the subroutine PhyImplTendency.

5 Surface energy balance and diffusion of heat in the soil

Prognostic equation of soil temperature is as follows:

$$C_g\frac{\partial T_g}{\partial t} = -\frac{\partial F_{g,h}}{\partial z} \quad (178)$$

$$F_{g,h} = F_{SR}+F_{LR}+F_h+LF_q \quad \text{for z} = 0 \quad (179)$$

$$F_{g,h} = -\kappa\frac{\partial T_g}{\partial z} \quad \text{for z} < 0 \quad (180)$$

where C_g is the specific heat per unit volume of the soil and κ is the diffusion coefficient for heat.

$$C_g \frac{T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}}{2\Delta t} = -\frac{F_{g,h,k+\frac{1}{2}}^{t+\Delta t} - F_{g,h,k-\frac{1}{2}}^{t+\Delta t}}{z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}}} \quad (181)$$

for $1 \le k \le k_{max} - 1$

$$F_{g,h,k+\frac{1}{2}} = -(TC)_{g,k+\frac{1}{2}} \left(T_{g,k+1} - T_{g,k}\right), \quad (182)$$

$$(TC)_{g,k+\frac{1}{2}} = \kappa_{g,k+\frac{1}{2}} \frac{1}{z_{k+1} - z_k} \quad (183)$$

for $k = 1$

$$F_{g,h,k-\frac{1}{2}} = F_{SR} + F_{LR} + F_{h,\frac{1}{2}} + LF_{q,\frac{1}{2}} \quad (184)$$

for $k = k_{max}$

$$F_{g,h,k+\frac{1}{2}} = 0 \quad (185)$$

ここで,

$$\kappa_{k+\frac{1}{2}} = \quad (186)$$

である.

しかし, 上記の式だけでは, 方程式の数よりも未知数 (大気温度, T, 地表面温度, T_s, 土壌温度, T_g) の数の方が多いために解けない. 従って, 以下の式を導入する.

$$F_{g,h,\frac{1}{2}} = -(TC)_{g,\frac{1}{2}} \left(T_{g,1} - T_s\right) \quad (187)$$

$$(TC)_{g,\frac{1}{2}} = \kappa_{g,\frac{1}{2}} \frac{1}{z_1 - 0} \quad (188)$$

$$\kappa_{\frac{1}{2}} = \quad (189)$$

この式を導入することは, 地表面に熱容量ゼロの仮想的な層が存在することと等価である. そこで, ここではこの考えを拡張し, 一様な温度 T_s を持ち, 単位面積当たりの熱容量が C_s である層が地表面直下にあると考えることにする. この層の熱収支の式は ... の式を拡張し, 以下のように書くことができる.

$$C_s \frac{\partial T_s}{\partial t} = -F_{SR} - F_{LR} - F_{h,\frac{1}{2}} - LF_{q,\frac{1}{2}} + F_{g,h,\frac{1}{2}} \quad (190)$$

$C_s = 0$ の場合には, 上に述べた ... の式を適応した場合に対応する.

この定式化の利点は, slub ocean の条件に適応できることである. $C_s \ne 0$, $F_{g,h,k-\frac{1}{2}} = 0$, $(TC)_{g,\frac{1}{2}} = 0$ の場合には, slub ocean に対応する. しかし, これは単に計算上 / モデル開発上の工夫である. ... よね?

この式を時間に関して離散化すると,

$$C_s \frac{T_s^{t+\Delta t} - T_s^{t-\Delta t}}{2\Delta t} = -F_{SR} - F_{LR} - F_{h,\frac{1}{2}} - LF_{q,\frac{1}{2}} + F_{g,h,\frac{1}{2}} \quad (191)$$

となる.

For the case with finite surface heat capacity,

$$\frac{C_s}{2\Delta t}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{192}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}^{t+\Delta t}-F_{h,\frac{1}{2}}^{t+\Delta t}+F_{h,\frac{1}{2}}^{t-\Delta t}-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{193}$$
$$+F_{g,h,\frac{1}{2}}^{t+\Delta t}-F_{g,h,\frac{1}{2}}^{t-\Delta t}+F_{g,h,\frac{1}{2}}^{t-\Delta t} \tag{194}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}^{t+\Delta t} \tag{195}$$
$$+C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}T_1^{t+\Delta t}-C_p(TC)_{h,\frac{1}{2}}T_s^{t+\Delta t} \tag{196}$$
$$-C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}T_1^{t-\Delta t}+C_p(TC)_{h,\frac{1}{2}}T_s^{t-\Delta t}-F_{h,\frac{1}{2}}^{t-\Delta t} \tag{197}$$
$$-LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{198}$$
$$-(TC)_{g,\frac{1}{2}}\left(T_{g,1}^{t+\Delta t}-T_s^{t+\Delta t}\right)+(TC)_{g,\frac{1}{2}}\left(T_{g,1}^{t-\Delta t}-T_s^{t-\Delta t}\right)+F_{g,h,\frac{1}{2}}^{t-\Delta t} \tag{199}$$
$$= -F_{SR}^{t+\Delta t} \tag{200}$$
$$-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-\frac{\partial F_{LR}}{\partial T_s}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right)-\frac{\partial F_{LR}}{\partial T_1}\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) \tag{201}$$
$$+C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right)-C_p(TC)_{h,\frac{1}{2}}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{202}$$
$$-(TC)_{g,\frac{1}{2}}\left(T_{g,1}^{t+\Delta t}-T_{g,1}^{t-\Delta t}\right)+(TC)_{g,\frac{1}{2}}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{203}$$
$$-F_{h,\frac{1}{2}}^{t-\Delta t} \tag{204}$$
$$-LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{205}$$
$$+F_{g,h,\frac{1}{2}}^{t-\Delta t} \tag{206}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,h,\frac{1}{2}}^{t-\Delta t} \tag{207}$$
$$+\left((TC)_{g,\frac{1}{2}}-C_p(TC)_{h,\frac{1}{2}}-\frac{\partial F_{LR}}{\partial T_s}\right)\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{208}$$
$$+\left(C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}-\frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) \tag{209}$$
$$-(TC)_{g,\frac{1}{2}}\left(T_{g,1}^{t+\Delta t}-T_{g,1}^{t-\Delta t}\right) \tag{210}$$

ただし, ここでは三重対角行列にするために, 潜熱フラックスは $t-\Delta t$ の時刻のものを使う. これより

$$(TC)_{g,\frac{1}{2}}\left(T_{g,1}^{t+\Delta t}-T_{g,1}^{t-\Delta t}\right) \tag{211}$$
$$+\left(\frac{C_s}{2\Delta t}-(TC)_{g,\frac{1}{2}}+C_p(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_s}\right)\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{212}$$
$$+\left(-C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) \tag{213}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,h,\frac{1}{2}}^{t-\Delta t} \tag{214}$$

for the case with finite heat capacity ($k=0$),

$$b_{k,k-1} = (TC)_{g,\frac{1}{2}} \tag{215}$$
$$b_{k,k} = \frac{C_s}{2\Delta t}-(TC)_{g,\frac{1}{2}}+C_p(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_s} \tag{216}$$

$$b_{k,k+1} = -C_p \frac{1}{P_1}(TC)_{h,\frac{1}{2}} + \frac{\partial F_{LR}}{\partial T_1} \quad (217)$$

$$g_{h,k} = -F_{SR}^{t+\Delta t} - F_{LR}\left(T_s^{t-\Delta t}, T_1^{t-\Delta t}\right) - F_{h,\frac{1}{2}}^{t-\Delta t} - LF_{q,\frac{1}{2}}^{t-\Delta t} + F_{g,h,\frac{1}{2}}^{t-\Delta t} \quad (218)$$

Diffusion equation of heat in the soil is expressed as follows,
for $2 \leq k \leq k_{max} - 1$

$$\frac{1}{2\Delta t} C_{g,k}\left(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}}\right)\left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}\right) \quad (219)$$

$$= -F_{g,h,k+\frac{1}{2}}^{t+\Delta t} + F_{g,h,k-\frac{1}{2}}^{t+\Delta t} \quad (220)$$

$$= -F_{g,h,k+\frac{1}{2}}^{t+\Delta t} + F_{g,h,k-\frac{1}{2}}^{t+\Delta t} + F_{g,h,k+\frac{1}{2}}^{t-\Delta t} - F_{g,h,k-\frac{1}{2}}^{t-\Delta t} - F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (221)$$

$$= +(TC)_{g,k+\frac{1}{2}}\left(T_{g,k+1}^{t+\Delta t} - T_{g,k}^{t+\Delta t}\right) - (TC)_{g,k-\frac{1}{2}}\left(T_{g,k}^{t+\Delta t} - T_{g,k-1}^{t+\Delta t}\right) \quad (222)$$

$$-(TC)_{g,k+\frac{1}{2}}\left(T_{g,k+1}^{t-\Delta t} - T_{g,k}^{t-\Delta t}\right) + (TC)_{g,k-\frac{1}{2}}\left(T_{g,k}^{t-\Delta t} - T_{g,k-1}^{t-\Delta t}\right) \quad (223)$$

$$-F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (224)$$

$$= +(TC)_{g,k-\frac{1}{2}}\left(T_{g,k-1}^{t+\Delta t} - T_{g,k-1}^{t-\Delta t}\right) - \left((TC)_{g,k-\frac{1}{2}} + (TC)_{g,k+\frac{1}{2}}\right)\left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}\right) \quad (225)$$

$$+(TC)_{g,k+\frac{1}{2}}\left(T_{g,k+1}^{t+\Delta t} - T_{g,k+1}^{t-\Delta t}\right) \quad (226)$$

$$-F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (227)$$

より,

$$-(TC)_{g,k-\frac{1}{2}}\left(T_{g,k-1}^{t+\Delta t} - T_{g,k-1}^{t-\Delta t}\right) \quad (228)$$

$$+\left\{\frac{1}{2\Delta t} C_{g,k}, \left(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}}\right) + (TC)_{g,k-\frac{1}{2}} + (TC)_{g,k+\frac{1}{2}}\right\}\left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}\right) \quad (229)$$

$$-(TC)_{g,k+\frac{1}{2}}\left(T_{g,k+1}^{t+\Delta t} - T_{g,k+1}^{t-\Delta t}\right) \quad (230)$$

$$= -\left(F_{g,h,k+\frac{1}{2}}^{t-\Delta t} - F_{g,h,k-\frac{1}{2}}^{t-\Delta t}\right) \quad (231)$$

for $k = 1$

$$T_{g,k-1} = T_s \quad (232)$$

として上記の式を使えば良い.
for $k = k_{max}$

$$\frac{1}{2\Delta t} C_{g,k}\left(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}}\right)\left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}\right) \quad (233)$$

$$= -F_{g,h,k+\frac{1}{2}}^{t+\Delta t} + F_{g,h,k-\frac{1}{2}}^{t+\Delta t} \quad (234)$$

$$= -F_{g,h,k+\frac{1}{2}}^{t+\Delta t} + F_{g,h,k-\frac{1}{2}}^{t+\Delta t} + F_{g,h,k+\frac{1}{2}}^{t-\Delta t} - F_{g,h,k-\frac{1}{2}}^{t-\Delta t} - F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (235)$$

$$= +0 - (TC)_{g,k-\frac{1}{2}}\left(T_{g,k}^{t+\Delta t} - T_{g,k-1}^{t+\Delta t}\right) \quad (236)$$

$$-0 + (TC)_{g,k-\frac{1}{2}}\left(T_{g,k}^{t-\Delta t} - T_{g,k-1}^{t-\Delta t}\right) \quad (237)$$

$$-F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (238)$$

$$= +(TC)_{g,k-\frac{1}{2}}\left(T_{g,k-1}^{t+\Delta t} - T_{g,k-1}^{t-\Delta t}\right) - (TC)_{g,k-\frac{1}{2}}\left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t-\Delta t}\right) \quad (239)$$

$$-F_{g,h,k+\frac{1}{2}}^{t-\Delta t} + F_{g,h,k-\frac{1}{2}}^{t-\Delta t} \quad (240)$$

より,

$$-(TC)_{g,k-\frac{1}{2}}\left(T_{g,k-1}^{t+\Delta t}-T_{g,k-1}^{t-\Delta t}\right) \tag{241}$$
$$+\left\{\frac{1}{2\Delta t}C_{g,k},\left(z_{k+\frac{1}{2}}-z_{k-\frac{1}{2}}\right)+(TC)_{g,k-\frac{1}{2}}\right\}\left(T_{g,k}^{t+\Delta t}-T_{g,k}^{t-\Delta t}\right) \tag{242}$$
$$= -\left(F_{g,h,k+\frac{1}{2}}^{t-\Delta t}-F_{g,h,k-\frac{1}{2}}^{t-\Delta t}\right) \tag{243}$$

for $1 \leq k \leq k_{max}-1$

$$b_{k,k-1} = -(TC)_{g,k-\frac{1}{2}} \tag{244}$$
$$b_{k,k} = \frac{1}{2\Delta t}C_{g,k},\left(z_{k+\frac{1}{2}}-z_{k-\frac{1}{2}}\right)+(TC)_{g,k-\frac{1}{2}}+(TC)_{g,k+\frac{1}{2}} \tag{245}$$
$$b_{k,k+1} = -(TC)_{g,k+\frac{1}{2}} \tag{246}$$

for $k = k_{max}$

$$b_{k,k-1} = -(TC)_{g,k-\frac{1}{2}} \tag{247}$$
$$b_{k,k} = \frac{1}{2\Delta t}C_{g,k},\left(z_{k+\frac{1}{2}}-z_{k-\frac{1}{2}}\right)+(TC)_{g,k-\frac{1}{2}} \tag{248}$$

6 Surface energy balance on the sea ice

Energy balance at the surface is as follows:

$$C_i\frac{\partial T_s}{\partial t} = -F_{SR}-F_{LR}-F_{h,\frac{1}{2}}-LF_{q,\frac{1}{2}}+F_g. \tag{249}$$

$$C_i\frac{T_s^{t+\Delta t}-T_s^{t-\Delta t}}{2\Delta t} = -F_{SR}-F_{LR}-F_{h,\frac{1}{2}}-LF_{q,\frac{1}{2}}+F_g. \tag{250}$$

where C_i is the heat capacity of sea ice.

$$\frac{C_i}{2\Delta t}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{251}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}^{t+\Delta t}-F_{h,\frac{1}{2}}^{t+\Delta t}+F_{h,\frac{1}{2}}^{t-\Delta t}-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,\frac{1}{2}}^{t+\Delta t}-F_{g,\frac{1}{2}}^{t-\Delta t}+F_{g,\frac{1}{2}}^{t-\Delta t} \tag{252}$$
$$= -F_{SR}^{t+\Delta t}-F_{LR}^{t+\Delta t} \tag{253}$$
$$+C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}T_1^{t+\Delta t}-C_p(TC)_{h,\frac{1}{2}}T_s^{t+\Delta t} \tag{254}$$
$$-C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}T_1^{t-\Delta t}+C_p(TC)_{h,\frac{1}{2}}T_s^{t-\Delta t}-F_{h,\frac{1}{2}}^{t-\Delta t} \tag{255}$$
$$-LF_{q,\frac{1}{2}}^{t-\Delta t} \tag{256}$$
$$-\frac{\kappa_i}{h_i}\left(T_s^{t+\Delta t}-T_o\right)+\frac{\kappa_i}{h_i}\left(T_s^{t-\Delta t}-T_o\right)+F_{g,\frac{1}{2}}^{t-\Delta t} \tag{257}$$
$$= -F_{SR}^{t+\Delta t} \tag{258}$$
$$-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-\frac{\partial F_{LR}}{\partial T_s}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right)-\frac{\partial F_{LR}}{\partial T_1}\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) \tag{259}$$
$$+C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right)-C_p(TC)_{h,\frac{1}{2}}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \tag{260}$$

$$
\begin{aligned}
& -\frac{\kappa_i}{h_i}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) && (261)\\
& -F_{h,\frac{1}{2}}^{t-\Delta t} && (262)\\
& -LF_{q,\frac{1}{2}}^{t-\Delta t} && (263)\\
& +F_{g,\frac{1}{2}}^{t-\Delta t} && (264)\\
= & -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,\frac{1}{2}}^{t-\Delta t} && (265)\\
& +\left(-C_p(TC)_{h,\frac{1}{2}}-\frac{\partial F_{LR}}{\partial T_s}-\frac{\kappa_i}{h_i}\right)\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) && (266)\\
& +\left(C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}-\frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) && (267)
\end{aligned}
$$

ただし, 水の質量の保存を満たすために, 潜熱フラックスは $t-\Delta t$ の時刻のものを使う. これより

$$
\begin{aligned}
& \left(\frac{C_i}{2\Delta t}+C_p(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_s}+\frac{\kappa_i}{h_i}\right)\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) && (268)\\
& \quad +\left(-C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_1}\right)\left(T_1^{t+\Delta t}-T_1^{t-\Delta t}\right) && (269)\\
& \quad = -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,\frac{1}{2}}^{t-\Delta t} && (270)
\end{aligned}
$$

$$
\begin{aligned}
b_{k,k-1} &= 0 && (271)\\
b_{k,k} &= \frac{C_i}{2\Delta t}+C_p(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_s}+\frac{\kappa_i}{h_i} && (272)\\
b_{k,k+1} &= -C_p\frac{P_{\frac{1}{2}}}{P_1}(TC)_{h,\frac{1}{2}}+\frac{\partial F_{LR}}{\partial T_1} && (273)\\
g_{h,k} &= -F_{SR}^{t+\Delta t}-F_{LR}\left(T_s^{t-\Delta t},T_1^{t-\Delta t}\right)-F_{h,\frac{1}{2}}^{t-\Delta t}-LF_{q,\frac{1}{2}}^{t-\Delta t}+F_{g,\frac{1}{2}}^{t-\Delta t} && (274)
\end{aligned}
$$

7 Difference equation for diffusion of water vapor

$$
\begin{aligned}
\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) &= F_{q,k+\frac{1}{2}}^{t+\Delta t}-F_{q,k-\frac{1}{2}}^{t+\Delta t} && (275)\\
&= F_{q,k+\frac{1}{2}}^{t+\Delta t}-F_{q,k-\frac{1}{2}}^{t+\Delta t}-F_{q,k+\frac{1}{2}}^{t-\Delta t}+F_{q,k-\frac{1}{2}}^{t-\Delta t}+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (276)
\end{aligned}
$$

for $2\le k\le k_{max}-1$

$$
\begin{aligned}
& \frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) && (277)\\
= & F_{q,k+\frac{1}{2}}^{t+\Delta t}-F_{q,k-\frac{1}{2}}^{t+\Delta t}-F_{q,k+\frac{1}{2}}^{t-\Delta t}+F_{q,k-\frac{1}{2}}^{t-\Delta t}+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (278)\\
= & -(TC)_{q,k+\frac{1}{2}}q_{k+1}^{t+\Delta t}+(TC)_{q,k+\frac{1}{2}}q_k^{t+\Delta t}+(TC)_{q,k-\frac{1}{2}}q_k^{t+\Delta t}-(TC)_{q,k-\frac{1}{2}}q_{k-1}^{t+\Delta t} && (279)\\
& +(TC)_{q,k+\frac{1}{2}}q_{k+1}^{t-\Delta t}-(TC)_{q,k+\frac{1}{2}}q_k^{t-\Delta t}-(TC)_{q,k-\frac{1}{2}}q_k^{t-\Delta t}+(TC)_{q,k-\frac{1}{2}}q_{k-1}^{t-\Delta t} && (280)\\
& +F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (281)\\
= & -(TC)_{q,k-\frac{1}{2}}\left(q_{k-1}^{t+\Delta t}-q_{k-1}^{t-\Delta t}\right) && (282)
\end{aligned}
$$

$$+(TC)_{q,k+\frac{1}{2}}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (283)$$
$$+(TC)_{q,k-\frac{1}{2}}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (284)$$
$$-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) \quad (285)$$
$$+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (286)$$
$$= -(TC)_{q,k-\frac{1}{2}}\left(q_{k-1}^{t+\Delta t}-q_{k-1}^{t-\Delta t}\right) \quad (287)$$
$$+\left((TC)_{q,k+\frac{1}{2}}+(TC)_{q,k-\frac{1}{2}}\right)\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (288)$$
$$-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) \quad (289)$$
$$+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (290)$$

より

$$-(TC)_{q,k-\frac{1}{2}}\left(q_{k-1}^{t+\Delta t}-q_{k-1}^{t-\Delta t}\right) \quad (291)$$
$$+\left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{q,k+\frac{1}{2}}+(TC)_{q,k-\frac{1}{2}}\right)\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (292)$$
$$-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) \quad (293)$$
$$= -\left(F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t}\right) \quad (294)$$

for $k=1$

$$\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (295)$$
$$= F_{q,k+\frac{1}{2}}^{t+\Delta t}-F_{q,k-\frac{1}{2}}^{t+\Delta t}-F_{q,k+\frac{1}{2}}^{t-\Delta t}+F_{q,k-\frac{1}{2}}^{t-\Delta t}+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (296)$$
$$= -(TC)_{q,k+\frac{1}{2}}q_{k+1}^{t+\Delta t}+(TC)_{q,k+\frac{1}{2}}q_k^{t+\Delta t} \quad (297)$$
$$+\epsilon(TC)_{q,k-\frac{1}{2}}q_k^{t+\Delta t}-\epsilon(TC)_{q,k-\frac{1}{2}}q_s^{*,t+\Delta t} \quad (298)$$
$$+(TC)_{q,k+\frac{1}{2}}q_{k+1}^{t-\Delta t}-(TC)_{q,k+\frac{1}{2}}q_k^{t-\Delta t} \quad (299)$$
$$-\epsilon(TC)_{q,k-\frac{1}{2}}q_k^{t-\Delta t}+\epsilon(TC)_{q,k-\frac{1}{2}}q_s^{*,t-\Delta t} \quad (300)$$
$$+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (301)$$
$$= (TC)_{q,k+\frac{1}{2}}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (302)$$
$$-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) \quad (303)$$
$$+\epsilon(TC)_{q,k-\frac{1}{2}}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (304)$$
$$-\epsilon(TC)_{q,k-\frac{1}{2}}\left(q_s^{*,t+\Delta t}-q_s^{*,t-\Delta t}\right) \quad (305)$$
$$+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (306)$$
$$= \left((TC)_{q,k+\frac{1}{2}}+\epsilon(TC)_{q,k-\frac{1}{2}}\right)\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) \quad (307)$$
$$-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) \quad (308)$$
$$-\epsilon(TC)_{q,k-\frac{1}{2}}\frac{\partial q_s^*}{\partial T_s}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \quad (309)$$
$$+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} \quad (310)$$

より

$$-\epsilon(TC)_{q,k-\frac{1}{2}}\frac{\partial q_s^*}{\partial T_s}\left(T_s^{t+\Delta t}-T_s^{t-\Delta t}\right) \quad (311)$$

$$
\begin{aligned}
&+\left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{q,k+\frac{1}{2}}+\epsilon(TC)_{q,k-\frac{1}{2}}\right)\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) && (312)\\
&-(TC)_{q,k+\frac{1}{2}}\left(q_{k+1}^{t+\Delta t}-q_{k+1}^{t-\Delta t}\right) && (313)\\
= \quad & -\left(F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t}\right) && (314)
\end{aligned}
$$

バケツモデルを使う時には (表現が不正確だが), 左辺第一項の係数はゼロ, 左辺第二項内の第三項 (ϵ を含む項) はゼロ.

for $k = k_{max}$

$$
\begin{aligned}
&\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) && (315)\\
= \quad & -F_{q,k-\frac{1}{2}}^{t+\Delta t}+F_{q,k-\frac{1}{2}}^{t-\Delta t}+F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (316)\\
= \quad & (TC)_{q,k-\frac{1}{2}}q_k^{t+\Delta t}-(TC)_{q,k-\frac{1}{2}}q_{k-1}^{t+\Delta t} && (317)\\
& -(TC)_{q,k-\frac{1}{2}}q_k^{t-\Delta t}+(TC)_{q,k-\frac{1}{2}}q_{k-1}^{t-\Delta t} && (318)\\
& +F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (319)\\
= \quad & -(TC)_{q,k-\frac{1}{2}}\left(q_{k-1}^{t+\Delta t}-q_{k-1}^{t-\Delta t}\right) && (320)\\
& +(TC)_{q,k-\frac{1}{2}}\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) && (321)\\
& +F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t} && (322)
\end{aligned}
$$

より

$$
\begin{aligned}
&-(TC)_{q,k-\frac{1}{2}}\left(q_{k-1}^{t+\Delta t}-q_{k-1}^{t-\Delta t}\right) && (323)\\
&+\left(-\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{q,k-\frac{1}{2}}\right)\left(q_k^{t+\Delta t}-q_k^{t-\Delta t}\right) && (324)\\
= \quad & -\left(F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t}\right) && (325)
\end{aligned}
$$

$$
\boldsymbol{C}\boldsymbol{x}_q = \boldsymbol{G}_q \qquad (326)
$$

$$
\begin{aligned}
\boldsymbol{x}_q &= \left(q_1^{t+\Delta t}-q_1^{t-\Delta t}, q_2^{t+\Delta t}-q_2^{t-\Delta t}, ..., q_{k_{max}}^{t+\Delta t}-q_{k_{max}}^{t-\Delta t}\right), && (327)\\
\boldsymbol{G}_q &= \left(g_{q,1}, g_{q,2}, ..., g_{q,k_{max}}\right), && (328)\\
g_{q,k} &= -\left(F_{q,k+\frac{1}{2}}^{t-\Delta t}-F_{q,k-\frac{1}{2}}^{t-\Delta t}\right) && (329)
\end{aligned}
$$

for $2 \le k \le k_{max}-1$

$$
\begin{aligned}
c_{k,k-1} &= -(TC)_{q,k-\frac{1}{2}} && (330)\\
c_{k,k} &= -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{q,k+\frac{1}{2}}+(TC)_{q,k-\frac{1}{2}} && (331)\\
c_{k,k+1} &= -(TC)_{q,k+\frac{1}{2}} && (332)
\end{aligned}
$$

for $k = 1$

$$
\begin{aligned}
c_{k,k-1} &= -\epsilon(TC)_{q,k-\frac{1}{2}}\frac{\partial q_s^*}{\partial T_s} && (333)\\
c_{k,k} &= -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}}-p_{k-\frac{1}{2}}}{g}+(TC)_{q,k+\frac{1}{2}}+\epsilon(TC)_{q,k-\frac{1}{2}} && (334)\\
c_{k,k+1} &= -(TC)_{q,k+\frac{1}{2}} && (335)
\end{aligned}
$$

for $k = k_{max}$

$$c_{k,k-1} = -(TC)_{q,k-\frac{1}{2}} \tag{336}$$

$$c_{k,k} = -\frac{1}{2\Delta t}\frac{p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}}{g} + (TC)_{q,k-\frac{1}{2}} \tag{337}$$

$c_{k,k-1}$, $c_{k,k}$, $c_{k,k+1}$ にかかっている C_p は L の間違いではないだろうか? もしくは, もともと L はいらないのでは?
修正点:

- xyza_QVapMtx(:,:,k,0) in the subroutine PhyImplGetMatrices,
- xyza_QvapMtx(:,:,k,-1), xyza_QvapMtx(:,:,k,0), xyza_QvapMtx(:,:,k,1) in the subroutine VerticalDiffusion,
- xyaa_SurfQVapMtx(:,:,1,0) in the subroutine SurfaceFlux.

熱の式と水蒸気の式は, 地面での熱収支を通して結合しているため, 同時に解く必要がある. 具体的には, 以下の 2 式

$$\boldsymbol{B}\boldsymbol{x}_h = \boldsymbol{G}_h \tag{338}$$

$$\boldsymbol{C}\boldsymbol{x}_q = \boldsymbol{G}_q \tag{339}$$

を結合し,

$$\boldsymbol{D}\boldsymbol{x}_{hq} = \boldsymbol{G}_{hq} \tag{340}$$

$$\begin{aligned}
\boldsymbol{x}_{hq} &= \left(q_{k_{max}}^{t+\Delta t} - q_{k_{max}}^{t-\Delta t}, ..., q_2^{t+\Delta t} - q_2^{t-\Delta t}, q_1^{t+\Delta t} - q_1^{t-\Delta t}, T_s^{t+\Delta t} - T_s^{t-\Delta t}, T_1^{t+\Delta t} - T_1^{t-\Delta t}, T_2^{t+\Delta t} - T_2^{t-\Delta t}, ..., T_{k_{max}}^{t+\Delta t}\right. \\
\boldsymbol{G}_{hq} &= \left(g_{q,k_{max}}, ..., g_{q,2}, g_{q,1}, g_{h,0}, g_{h,1}, g_{h,2}, ..., g_{h,k_{max}}\right), \\
g_{q,k} &= -\left(F_{q,k+\frac{1}{2}}^{t-\Delta t} - F_{q,k-\frac{1}{2}}^{t-\Delta t}\right) \\
g_{h,k} &= -\left(F_{h,k+\frac{1}{2}}^{t-\Delta t} - F_{h,k-\frac{1}{2}}^{t-\Delta t}\right)
\end{aligned}$$

for $k < 0$,

$$d_{-k,k+1} = c_{k,k-1} \tag{345}$$

$$d_{-k,k} = c_{k,k} \tag{346}$$

$$d_{-k,k-1} = c_{k,k+1} \tag{347}$$

else

$$d_{k,k-1} = b_{k,k-1} \tag{348}$$

$$d_{k,k} = b_{k,k} \tag{349}$$

$$d_{k,k+1} = b_{k,k+1} \tag{350}$$

ついでに.
subroutine PhyImplFluxCorrect の中の xyr_QVapFlux の修正の LC_p をかけるのは間違っている

のでは? せめて LC_p で割ると値はまともになる? それとも今のままでいいのか? 確かに何かしないといけないのかもしれないが. いや, L をかけるだけにするといいのかな? DQVapDt の単位は 1 なので, transfer coefficient に L をかけないと, 熱フラックスの単位である QVapFlux と合わない. でも C_p はいらないような気がする.
いずれにしろ, ここで修正された QVapFlux は計算には使われないので計算結果は問題ないが.

メモ:
いたるところに L の代わりに C_p がかかっているが, 最後に subroutine PhyImplTendency の中で xyz_DQVapDt に $CpDry/Latentheat$ をかけることで合わせている. 意味がわからん.
結果として, 海に対しては, 表面温度が変わらない場合の行列の作り方を除くと答えは一緒かも. 陸に対しては, 色々なところで問題が生じていると思う.

8 Correction of thermal balance due to snow melt

地表面, および土壌一層目の熱伝導方程式は,

$$F_g^{n+1} = -\kappa\frac{T_s^{n+1}-T_{g,1}^{n+1}}{z_{\frac{1}{2}}-z_1} \tag{351}$$

$$= F_s^{n+1}+F_L^{n+1}+F_c^{n+1}+LF_q^{n+1}+F_{SM}^{n+1} \tag{352}$$

$$C_{g,1}\frac{T_{g,1}^{n+1}-T_{g,1}^{n-1}}{2\Delta t} = -\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\left\{F_s^{n+1}+F_L^{n+1}+F_c^{n+1}+LF_q^{n+1}+F_{SM}^{n+1}-F_g^{n+1}\right\}, \tag{353}$$

であり, ここで, $F_g^{n+1}, F_s^{n+1}, F_L^{n+1}, F_c^{n+1}, LF_q^{n+1}, F_{SM}^{n+1}$ はそれぞれ, 地下への熱伝導フラックス, 短波放射フラックス, 長波放射フラックス, 顕熱フラックス, 潜熱フラックス, 融雪による熱フラックスである.

$F_s^{n+1}, F_c^{n+1}, LF_q^{n+1}, F_g^{n+1}$ は, 前節までの方法で連立一次方程式を解くことで得られた値を用いることとし, 地表面温度 $T_s^{n+1}, T_{g,1}^{n+1}$ を介して F_L^{n+1} を調節することで融雪量, すなわち F_{SM}^{n+1} を求める. ここで,

$$F_L^{n+1} = F_L^{n-1}+\frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right)+\frac{\partial F_L}{\partial T_s}\left(T_s^{n+1}-T_s^{n-1}\right) \tag{354}$$

であるから, (...), (...) より,

$$-\kappa\frac{T_s^{n+1}-T_{g,1}^{n+1}}{z_{\frac{1}{2}}-z_1} = F_s^{n+1}+F_L^{n-1}+\frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right)+\frac{\partial F_L}{\partial T_s}\left(T_s^{n+1}-T_s^{n-1}\right) +F_c^{n+1}+LF_q^{n+1}+F_{SM}^{n+1}, \tag{355}$$

$$C_{g,1}\frac{T_{g,1}^{n+1}-T_{g,1}^{n-1}}{2\Delta t} = -\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\left\{F_s^{n+1}+F_L^{n-1}+\frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right)+\frac{\partial F_L}{\partial T_s}\left(T_s^{n+1}-T_s^{n-1}\right) +F_c^{n+1}+LF_q^{n+1}+F_{SM}^{n+1}-F_g^{n+1}\right\}, \tag{356}$$

となる.

これら 2 つの方程式に対して, 未知数は $T_s^{n+1}, T_{g,1}^{n+1}, F_{SM}^{n+1}$ の 3 つである. そこでまず, 積雪すべてが解けた場合を想定し, $T_s^{n+1}, T_{g,1}^{n+1}$ を求めることにする. それによって得られた $T_s^{n+1}, T_{g,1}^{n+1}$ が不適切であった場合 ($T_{g,1}^{n+1} < T_{cond}$) には, $T_{g,1}^{n+1} = T_{cond}$ として, T_s^{n+1}, F_{SM}^{n+1} を求める.

8.1 積雪がすべて解ける場合

このとき, $F_{SM}^{n+1} = \frac{M_{\mathrm{snow}}}{2\Delta t}$ であり, T_s^{n+1}, $T_{g,1}^{n+1}$ が未知数である. ここで, M_{snow} は積雪量である. (...) より,

$$-\frac{\kappa}{z_{\frac{1}{2}}-z_1}T_s^{n+1} + \frac{\kappa}{z_{\frac{1}{2}}-z_1}T_{g,1}^{n+1} = F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) + \frac{\partial F_L}{\partial T_s}T_s^{n+1} - \frac{\partial F_L}{\partial T_s}T_s^{n-1} \tag{357}$$

$$+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1}, \tag{358}$$

$$\frac{\kappa}{z_{\frac{1}{2}}-z_1}T_{g,1}^{n+1} = F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) + \frac{\partial F_L}{\partial T_s}T_s^{n+1} - \frac{\partial F_L}{\partial T_s}T_s^{n-1} \tag{359}$$

$$+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1} \tag{360}$$

$$+\frac{\kappa}{z_{\frac{1}{2}}-z_1}T_s^{n+1}, \tag{361}$$

$$T_{g,1}^{n+1} = \left(1+\frac{z_{\frac{1}{2}}-z_1}{\kappa}\frac{\partial F_L}{\partial T_s}\right)T_s^{n+1} \tag{362}$$

$$+\frac{z_{\frac{1}{2}}-z_1}{\kappa}\left\{F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) - \frac{\partial F_L}{\partial T_s}T_s^{n-1}\right. \tag{363}$$

$$\left.+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1}\right\}, \tag{364}$$

$$T_{g,1}^{n+1} = \left(1+\frac{z_{\frac{1}{2}}-z_1}{\kappa}\frac{\partial F_L}{\partial T_s}\right)T_s^{n+1} \tag{365}$$

$$+\frac{z_{\frac{1}{2}}-z_1}{\kappa}\alpha \tag{366}$$

ここで,

$$\alpha = F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) - \frac{\partial F_L}{\partial T_s}T_s^{n-1} \tag{367}$$

$$+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1} \tag{368}$$

である. (...) を (...) に代入すると,

$$\frac{C_{g,1}}{2\Delta t}\left\{\left(1+\frac{z_{\frac{1}{2}}-z_1}{\kappa}\frac{\partial F_L}{\partial T_s}\right)T_s^{n+1} + \frac{z_{\frac{1}{2}}-z_1}{\kappa}\alpha\right\} - \frac{C_{g,1}}{2\Delta t}T_{g,1}^{n-1}$$

$$= -\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\left\{F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) + \frac{\partial F_L}{\partial T_s}\left(T_s^{n+1}-T_s^{n-1}\right)\right.$$

$$\left.+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1} - F_g^{n+1}\right\}, \tag{369}$$

$$= -\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\left\{F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) + \frac{\partial F_L}{\partial T_s}T_s^{n+1} - \frac{\partial F_L}{\partial T_s}T_s^{n-1}\right.$$

$$\left.+F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1}\right\}, \tag{370}$$

$$+\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}F_g^{n+1}, \tag{371}$$

$$= -\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\frac{\partial F_L}{\partial T_s}T_s^{n+1} \tag{372}$$

$$-\frac{1}{z_{\frac{1}{2}}-z_{\frac{3}{2}}}\left\{F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1}\left(T_1^{n+1}-T_1^{n-1}\right) - \frac{\partial F_L}{\partial T_s}T_s^{n-1}\right.$$

$$
\begin{aligned}
& +F_c^{n+1} + LF_q^{n+1} + F_{SM}^{n+1} \Big\}, && (373)\\
& + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1}, && (374)\\
= \quad & - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\partial F_L}{\partial T_s} T_s^{n+1} && (375)\\
& - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \alpha && (376)\\
& + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1}, && (377)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{C_{g,1}}{2\Delta t} \left(1 + \frac{z_{\frac{1}{2}} - z_1}{\kappa} \frac{\partial F_L}{\partial T_s} \right) T_s^{n+1} + \frac{C_{g,1}}{2\Delta t} \frac{z_{\frac{1}{2}} - z_1}{\kappa} \alpha - \frac{C_{g,1}}{2\Delta t} T_{g,1}^{n-1} \\
& \quad = \quad - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\partial F_L}{\partial T_s} T_s^{n+1} - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \alpha + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1}, \qquad (378)\\
& \left\{ \frac{C_{g,1}}{2\Delta t} \left(1 + \frac{z_{\frac{1}{2}} - z_1}{\kappa} \frac{\partial F_L}{\partial T_s} \right) + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\partial F_L}{\partial T_s} \right\} T_s^{n+1} \\
& \quad = \quad - \left(\frac{C_{g,1}}{2\Delta t} \frac{z_{\frac{1}{2}} - z_1}{\kappa} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \right) \alpha + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} + \frac{C_{g,1}}{2\Delta t} T_{g,1}^{n-1} \qquad (379)\\
& T_s^{n+1} \quad = \quad \left\{ \frac{C_{g,1}}{2\Delta t} \left(1 + \frac{z_{\frac{1}{2}} - z_1}{\kappa} \frac{\partial F_L}{\partial T_s} \right) + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\partial F_L}{\partial T_s} \right\}^{-1} \\
& \qquad \left\{ - \left(\frac{C_{g,1}}{2\Delta t} \frac{z_{\frac{1}{2}} - z_1}{\kappa} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \right) \alpha + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} + \frac{C_{g,1}}{2\Delta t} T_{g,1}^{n-1} \right\} \qquad (380)
\end{aligned}
$$

8.2 積雪がすべて解けない場合

このとき, $T_{g,1}^{n+1} = T_{cond}$ であり, F_{SM}, T_s^{n+1} が未知数である.
(...) より,

$$
\begin{aligned}
F_{SM}^{n+1} \quad = \quad & -\kappa \frac{T_s^{n+1} - T_{g,1}^{n+1}}{z_{\frac{1}{2}} - z_1} - F_s^{n+1} - F_L^{n-1} - \frac{\partial F_L}{\partial T_1} \left(T_1^{n+1} - T_1^{n-1} \right) - \frac{\partial F_L}{\partial T_s} \left(T_s^{n+1} - T_s^{n-1} \right) \\
& - F_c^{n+1} - LF_q^{n+1} \qquad (381)
\end{aligned}
$$

(...) に代入すると,

$$
\begin{aligned}
C_{g,1} \frac{T_{g,1}^{n+1} - T_{g,1}^{n-1}}{2\Delta t} \quad = \quad & - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \Big\{ F_s^{n+1} + F_L^{n-1} + \frac{\partial F_L}{\partial T_1} \left(T_1^{n+1} - T_1^{n-1} \right) + \frac{\partial F_L}{\partial T_s} \left(T_s^{n+1} - T_s^{n-1} \right) \\
& + F_c^{n+1} + LF_q^{n+1} \qquad (382)\\
& - \kappa \frac{T_s^{n+1} - T_{g,1}^{n+1}}{z_{\frac{1}{2}} - z_1} - F_s^{n+1} - F_L^{n-1} - \frac{\partial F_L}{\partial T_1} \left(T_1^{n+1} - T_1^{n-1} \right) - \frac{\partial F_L}{\partial T_s} \left(T_s^{n+1} - T_s^{n-1} \right) \\
& - F_c^{n+1} - LF_q^{n+1} \qquad (383)\\
& - F_g^{n+1} \Big\}, \qquad (384)
\end{aligned}
$$

$$= -\frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \left\{ -\kappa \frac{T_s^{n+1} - T_{g,1}^{n+1}}{z_{\frac{1}{2}} - z_1} - F_g^{n+1} \right\} \tag{385}$$

$$= \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \kappa \frac{T_s^{n+1} - T_{g,1}^{n+1}}{z_{\frac{1}{2}} - z_1} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} \tag{386}$$

$$= \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\kappa}{z_{\frac{1}{2}} - z_1} T_s^{n+1} - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\kappa}{z_{\frac{1}{2}} - z_1} T_{g,1}^{n+1} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} \tag{387}$$

$$\frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\kappa}{z_{\frac{1}{2}} - z_1} T_s^{n+1} = C_{g,1} \frac{T_{g,1}^{n+1} - T_{g,1}^{n-1}}{2\Delta t} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\kappa}{z_{\frac{1}{2}} - z_1} T_{g,1}^{n+1} - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} \tag{388}$$

$$T_s^{n+1} = \left(z_{\frac{1}{2}} - z_{\frac{3}{2}} \right) \frac{z_{\frac{1}{2}} - z_1}{\kappa} \left(C_{g,1} \frac{T_{g,1}^{n+1} - T_{g,1}^{n-1}}{2\Delta t} + \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} \frac{\kappa}{z_{\frac{1}{2}} - z_1} T_{g,1}^{n+1} - \frac{1}{z_{\frac{1}{2}} - z_{\frac{3}{2}}} F_g^{n+1} \right) \tag{389}$$