NumRu::GPhys::EP_Flux
数理ドキュメント

地球流体電脳倶楽部

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第1章　はじめに

NumRu::GPhys::EP_Flux は Eliassen-Palm フラックス (EP フラックス) および残差循環を計算するメソッドを集めたモジュールである. 現状では, 鉛直座標として対数圧力座標を用いた球座標系におけるプリミティブ方程式 (準地衡風近似をしない) EP フラックスのためのメソッドだけが用意されている. 将来的には Plumb フラックスや Takaya-Nakamura フラックスを計算するメソッドもサポートする予定である. 本ドキュメントでは NumRu::GPhys::EP_Flux で使用される数式の解説と各メソッドの概説を行う. なお, NumRu::GPhys::EP_Flux では, 微分演算のために, 別モジュール NumRu::Derivative および NumRu::GPhys::Derivative で定義されるメソッドを使用している. 微分演算メソッドに関する詳細はそれぞれのモジュールのドキュメントを参照されたい.

第2章 NumRu::GPhys::EP_Flux で計算される緒量

本章では NumRu::GPhys::EP_Flux で定義される緒量の解説を行う．数理モデルは Andrews *et al.*(1987) の第 3 章に基づく．

2.1 系の設定

球面上の大気を考える．大気の厚さは水平方向の広がりに比べ薄く, 鉛直方向に静水圧平衡が成り立つものとする．緯度経度座標系を用い, 経度 λ 軸を東向き, 緯度 ϕ 軸を北向きに正をとる．鉛直座標には対数圧力座標 z^*

$$z^* \quad = \quad -H\ln(p/p_s), \qquad H = \frac{R_d T_s}{g_0} \tag{2.1}$$

を用いる．ここで H はスケールハイト, R_d は乾燥空気の気体定数 (普遍気体定数を R, 乾燥空気の分子量を w とすると $R_d = R/w$), T_s は標準参照温度 (定数), g_0 は地表面における重力加速度 (定数), p は圧力, p_s は参照圧力である．p_s として地表面圧力の代表値 (定数) を用いる．

2.2 EP フラックス

本モジュールでは惑星半径と後述の ρ_s で規格化した EP フラックス (以降, 規格化した EP フラックス) を計算, 出力する. 規格化した EP フラックスは

$$\hat{F}_\phi \equiv \sigma \cos\phi \left(\frac{\partial \overline{u}}{\partial z^*} \frac{\overline{v'\theta'}}{\frac{\partial \overline{\theta}}{\partial z^*}} - \overline{u'v'} \right), \tag{2.2a}$$

$$\hat{F}_{z^*} \equiv \sigma \cos\phi \left(\left[f - \frac{1}{a\cos\phi} \frac{\partial \overline{u}\cos\phi}{\partial \phi} \right] \frac{\overline{v'\theta'}}{\frac{\partial \overline{\theta}}{\partial z^*}} - \overline{u'w'} \right) \tag{2.2b}$$

と定義される. ここで $\hat{F}_\phi$, $\hat{F}_{z^*}$ はそれぞれ規格化された EP フラックスの ϕ 成分, z^* 成分である. $\overline{\bullet}$ は東西オイラー平均量, $\bullet'$ は東西オイラー平均量からのずれを表す. u, v, w はそれぞれ東西風速, 南北風速, 対数圧力速度で

$$(u, v, w) \equiv \left(a\cos\phi \frac{d\lambda}{dt}, a\frac{d\phi}{dt}, \frac{dz^*}{dt} \right)$$

と定義される. θ は温位, a は惑星半径 (定数) である. σ は

$$\sigma \equiv \frac{\rho_0}{\rho_s} = \exp\left(\frac{-z^*}{H} \right), \tag{2.3}$$

である. ただし, ρ_0 は基本場の密度で

$$\rho_0(z^*) \equiv \rho_s e^{-z^*/H}, \qquad \rho_s \equiv p_s/RT_s$$

である. f はコリオリパラメータで

$$f = 2\Omega \sin\phi = \frac{4\pi}{T_{rot}} \sin\phi \tag{2.4}$$

と定義される. Ω は自転角速度, T_{rot} は自転周期である. 本モジュールでは, 自転角速度を変更するためには T_{rot} の値を与える仕様になっている.

一方, Andrews *et al.* (1987) で示されている EP フラックスは次のように定義される

$$F_\phi = \rho_0 a \cos\phi \left(\frac{\partial \overline{u}}{\partial \overline{z^*}} \frac{\overline{v'\theta'}}{\frac{\partial \overline{\theta}}{\partial z^*}} - \overline{u'v'} \right) \tag{2.5a}$$

$$F_z^* = \rho_0 a \cos\phi \left(\left[f - \frac{\frac{\partial \overline{u}\cos\phi}{\partial \phi}}{a\cos\phi} \right] \frac{\overline{v'\theta'}}{\frac{\partial \overline{\theta}}{\partial z^*}} - \overline{u'w'} \right). \tag{2.5b}$$

ここで F_ϕ, F_{z^*} はそれぞれ EP フラックスの ϕ 成分, z^* 成分である. F_y, F_z^* と $\hat{F}_y, \hat{F}_z^*$ は以下のように関係付けられる.

$$(F_y, F_z^*) = a\rho_s(\hat{F}_y, \hat{F_{z^*}}) \tag{2.6}$$

2.3 残差循環

残差循環 $(0, \overline{v}^*, \overline{w}^*)$ は以下の形で定義される.

$$\begin{aligned}\overline{v}^* &\equiv \overline{v} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right) \\ &= \overline{v} - \frac{1}{\sigma}\frac{\partial}{\partial z^*}\left(\sigma \frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right) \qquad (2.7a)\\ \overline{w}^* &\equiv \overline{w} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi \frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right) \qquad (2.7b)\end{aligned}$$

2.4 平均東西流の式

規格化した EP フラックスを用いると, TEM 系における u の式は以下のようになる.

$$\frac{\partial\overline{u}}{\partial t} + \overline{v}^*\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{u}\cos\phi) - f\right] + \overline{w}^*\frac{\partial\overline{u}}{\partial z^*} - \overline{X} = \frac{1}{\sigma\cos\phi}\nabla\cdot\hat{\boldsymbol{F}}. \qquad (2.8)$$

2.5 子午面上の発散演算子

子午面における発散演算子は, $\boldsymbol{F}$ を任意のベクトルした時に以下の形で定義される.

$$\nabla\cdot\boldsymbol{F} = \frac{1}{a\cos\phi}\frac{\partial(\cos\phi F_\phi)}{\partial\phi} + \frac{\partial F_{z^*}}{\partial z^*} \qquad (2.9)$$

2.6 質量流線関数

残差循環の質量流線関数 Ψ^* を

$$\sigma\overline{v}^* = -g\frac{1}{2\pi a\cos\phi}\frac{\partial\Psi^*}{\partial z^*}, \qquad (2.10a)$$

$$\sigma\overline{w}^* = g\frac{1}{2\pi a^2\cos\phi}\frac{\partial\Psi^*}{\partial\phi} \qquad (2.10b)$$

と定義する. 上式を積分して Ψ^* を求めるために, 本モジュールでは (2.1) を使用して対数圧力座標 (z^*) 系から圧力座標 (p) 系へ

$$\frac{\partial}{\partial z^*}\Psi^* = -\frac{p}{H}\frac{\partial}{\partial p}\Psi^* \tag{2.11}$$

と変換し, 大気上端 ($p = 0$) において $\Psi^* = 0$ として積分し

$$\Psi^*(\theta, p) = \frac{2\pi a \cos\phi}{g}\int_0^p \overline{v}^*\, dp \tag{2.12}$$

と質量流線関数を導いている.

2.7 変数変換

EP_Flux モジュールでは与えられたデータに応じて変数変換を施す場合がある. その変換は以下のように行う.

入力されるデータの鉛直軸が気圧軸であった場合, 以下の関係式を用いて高度軸に変換し, 計算を行う.

$$z^* = -H\log\left(\frac{p}{p_{00}}\right), \tag{2.13a}$$

$$p = p_{00}\exp\left(-\frac{z^*}{H}\right) \tag{2.13b}$$

ここで p は圧力, p_{00} は地表面参考気圧 (定数) である.

入力が θ や w でなく, 気温 T, 圧力「速度」$\omega \equiv Dp/Dt$ の場合はそれぞれを元に w, θ を求める必要がある. 本モジュールでは以下の式を用いて w, θ を求める.

$$w = -\omega H/p \tag{2.14}$$

$$\theta = T\left(\frac{p_{00}}{p}\right)^{\kappa}, \kappa = R/C_p \tag{2.15}$$

ここで R, C_p はそれぞれ乾燥空気の気体定数および定圧比熱である.

付 録A　プリミティブ方程式系と変形オイラー平均の復習

本章では変形オイラー平均方程式系と EP フラックス および残差循環の関係を確認する．まず対数圧力座標系を用いた球面上の３次元プリミティブ方程式を提示する．次いでそのオイラー平均および変形オイラー平均方程式系を導出する．最後に変形オイラー平均方程式に基づき EP フラックスおよび残差循環を定義する．

1.1　球面上の対数圧力座標系におけるプリミティブ方程式

球面上の対数圧力座標系におけるプリミティブ方程式は以下の通りである．ここでは Andrews *et al.* (1987) の (3.1.3) 式を参考にした．

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{a\cos\phi}\frac{\partial\Phi}{\partial\lambda} = X, \tag{A.1a}$$

$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{a}\frac{\partial\Phi}{\partial\phi} = Y, \tag{A.1b}$$

$$\frac{\partial\Phi}{\partial z^*} = \frac{R\theta e^{-\kappa z^*/H}}{H}, \tag{A.1c}$$

$$\frac{1}{a\cos\phi}\left[\frac{\partial u}{\partial\lambda} + \left(\frac{\partial v\cos\phi}{\partial\phi}\right)\right] + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0 w) = 0, \tag{A.1d}$$

$$\frac{d\theta}{dt} = Q, \tag{A.1e}$$

ここで Φ はジオポテンシャルハイト, X, Y はそれぞれ外力の λ 成分 と ϕ 成分, $\kappa = R_d/c_p$ (c_p は等圧比熱) である. Q は非断熱加熱項で,

$$Q = \frac{J}{C_p} e^{\kappa z^*/H}$$

である. J は単位質量あたりの非断熱加熱率である. ここで明記した以外の変数の定義については第 2.1 節, 第 2.2 節 をを参照のこと.

1.2 オイラー平均方程式系

ある物理量 A について, ϕ, z^*, t を固定して東西方向にとった平均

$$\overline{A}(\phi, z^*, t) \equiv \frac{1}{2\pi}\int_0^{2\pi} A(\lambda, \phi, z^*, t)\, d\lambda \tag{A.2}$$

をオイラー平均と呼ぶ. オイラー平均からのずれを A' とすると

$$A' = A - \overline{A} \tag{A.3}$$

である. 定義により, $\overline{A'} = 0$, $\partial\overline{A}/\partial\lambda = 0$ となる.

(A.1) 中の各量をオイラー平均とそこからのずれに分けて書くと

$$\begin{aligned}
&\frac{\partial}{\partial t}(\overline{u}+u') + \frac{\overline{u}+u'}{a\cos\phi}\frac{\partial}{\partial\lambda}(\overline{u}+u') + \frac{\overline{v}+v'}{a}\frac{\partial}{\partial\phi}(\overline{u}+u') + (\overline{w}+w')\frac{\partial}{\partial z^*}(\overline{u}+u') \\
&\qquad - \left[f + \frac{\tan\phi}{a}(\overline{u}+u')\right](\overline{v}+v') + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(\overline{\Phi}+\Phi') = \overline{X} + X', && \text{(A.4a)} \\
&\frac{\partial}{\partial t}(\overline{v}+v') + \frac{\overline{u}+u'}{a\cos\phi}\frac{\partial}{\partial\lambda}(\overline{v}+v') + \frac{\overline{v}+v'}{a}\frac{\partial}{\partial\phi}(\overline{v}+v') + (\overline{w}+w')\frac{\partial}{\partial z^*}(\overline{v}+v') \\
&\qquad + \left[f + \frac{\tan\phi}{a}(\overline{u}+u')\right](\overline{u}+u') + \frac{1}{a}\frac{\partial}{\partial\phi}(\overline{\Phi}+\Phi') = \overline{Y} + Y', && \text{(A.4b)} \\
&\frac{\partial}{\partial z^*}(\overline{\Phi}+\Phi') = \frac{Re^{-\kappa z^*/H}}{H}(\overline{\theta}+\theta'), && \text{(A.4c)} \\
&\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(\overline{u}+u') + \frac{\partial}{\partial\phi}\{(\overline{v}+v')\cos\phi\}\right] + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}[\rho_0(\overline{w}+w')] = 0, && \text{(A.4d)} \\
&\frac{\partial}{\partial t}(\overline{\theta}+\theta') + \frac{\overline{u}+u'}{a\cos\phi}\frac{\partial}{\partial\lambda}(\overline{\theta}+\theta') + \frac{\overline{v}+v'}{a}\frac{\partial}{\partial\phi}(\overline{\theta}+\theta') + (\overline{w}+w')\frac{\partial}{\partial z^*}(\overline{\theta}+\theta') \\
&\qquad = \overline{Q} + Q' && \text{(A.4e)}
\end{aligned}$$

となる．上記を変形して, 左辺に平均量と平均量同士の積の項を, 右辺にそれ以外の項をまとめると

$$
\begin{aligned}
&\frac{\partial \overline{u}}{\partial t} + \frac{\overline{u}}{a\cos\phi}\frac{\partial \overline{u}}{\partial \lambda} + \frac{\overline{v}}{a}\frac{\partial \overline{u}}{\partial \phi} + \overline{w}\frac{\partial \overline{u}}{\partial z^*} - f\overline{v} - \frac{\tan\phi}{a}\overline{u}\,\overline{v} + \frac{1}{a\cos\phi}\frac{\partial \overline{\Phi}}{\partial \lambda} - \overline{X} \\
&\quad = -\frac{\partial u'}{\partial t} - \frac{\overline{u}}{a\cos\phi}\frac{\partial u'}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial \overline{u}}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial u'}{\partial \lambda} \\
&\qquad - \frac{\overline{v}}{a}\frac{\partial u'}{\partial \phi} - \frac{v'}{a}\frac{\partial \overline{u}}{\partial \phi} - \frac{v'}{a}\frac{\partial u'}{\partial \phi} - \overline{w}\frac{\partial u'}{\partial z^*} - w'\frac{\partial \overline{u}}{\partial z^*} - w'\frac{\partial u'}{\partial z^*} + fv' \\
&\qquad + \frac{\tan\phi}{a}\overline{u}v' + \frac{\tan\phi}{a}u'\overline{v} + \frac{\tan\phi}{a}u'v' - \frac{1}{a\cos\phi}\frac{\partial \Phi'}{\partial \lambda} + X', \qquad \text{(A.5a)}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \overline{v}}{\partial t} + \frac{\overline{u}}{a\cos\phi}\frac{\partial \overline{v}}{\partial \lambda} + \frac{\overline{v}}{a}\frac{\partial \overline{v}}{\partial \phi} + \overline{w}\frac{\partial \overline{v}}{\partial z^*} + f\overline{u} + \frac{\tan\phi}{a}(\overline{u})^2 + \frac{1}{a}\frac{\partial \overline{\Phi}}{\partial \phi} - \overline{Y} \\
&\quad = -\frac{\partial v'}{\partial t} - \frac{\overline{u}}{a\cos\phi}\frac{\partial v'}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial \overline{v}}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial v'}{\partial \lambda} \\
&\qquad - \frac{\overline{v}}{a}\frac{\partial v'}{\partial \phi} - \frac{v'}{a}\frac{\partial \overline{v}}{\partial \phi} - \frac{v'}{a}\frac{\partial v'}{\partial \phi} - \overline{w}\frac{\partial v'}{\partial z^*} - w'\frac{\partial \overline{v}}{\partial z^*} - w'\frac{\partial v'}{\partial z^*} - fu' \\
&\qquad - 2\frac{\tan\phi}{a}\overline{u}u' - \frac{\tan\phi}{a}(u')^2 - \frac{1}{a\cos\phi}\frac{\partial \Phi'}{\partial \phi} + Y', \qquad \text{(A.5b)}
\end{aligned}
$$

$$
\frac{\partial \overline{\Phi}}{\partial z^*} - \frac{Re^{-\kappa z^*/H}}{H}\overline{\theta} = -\frac{\partial \Phi'}{\partial z^*} + \frac{Re^{-\kappa z^*/H}}{H}\theta', \qquad \text{(A.5c)}
$$

$$
\begin{aligned}
&\frac{1}{a\cos\phi}\left[\frac{\partial \overline{u}}{\partial \lambda} + \frac{\partial}{\partial \phi}(\overline{v}\cos\phi)\right] + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w}) \\
&\quad = -\frac{1}{a\cos\phi}\left[\frac{\partial u'}{\partial \lambda} + \frac{\partial}{\partial \phi}(v'\cos\phi)\right] - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0 w'), \qquad \text{(A.5d)}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \overline{\theta}}{\partial t} + \frac{\overline{u}}{a\cos\phi}\frac{\partial \overline{\theta}}{\partial \lambda} + \frac{\overline{v}}{a}\frac{\partial \overline{\theta}}{\partial \phi} + \overline{w}\frac{\partial \overline{\theta}}{\partial z^*} - \overline{Q} \\
&\quad = -\frac{\partial \theta'}{\partial t} - \frac{\overline{u}}{a\cos\phi}\frac{\partial \theta'}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial \overline{\theta}}{\partial \lambda} - \frac{u'}{a\cos\phi}\frac{\partial \theta'}{\partial \lambda} \\
&\qquad - \frac{\overline{v}}{a}\frac{\partial \theta'}{\partial \phi} - \frac{v'}{a}\frac{\partial \overline{\theta}}{\partial \phi} - \frac{v'}{a}\frac{\partial \theta'}{\partial \phi} - \overline{w}\frac{\partial \theta'}{\partial z^*} - w'\frac{\partial \overline{\theta}}{\partial z^*} - w'\frac{\partial \theta'}{\partial z^*} + Q' \qquad \text{(A.5e)}
\end{aligned}
$$

と書ける. (A.5) をオイラー平均すると,

$$
\begin{aligned}
&\frac{\partial\overline{u}}{\partial t}+\frac{1}{a}\overline{v}\frac{\partial\overline{u}}{\partial\phi}+\overline{w}\frac{\partial\overline{u}}{\partial z^*}-f\overline{v}-\frac{\tan\phi}{a}\overline{u}\,\overline{v}-\overline{X}\\
&\quad=-\frac{1}{a\cos\phi}\overline{u'\frac{\partial u'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial u'}{\partial\phi}}-\overline{w'\frac{\partial u'}{\partial z^*}}+\frac{\tan\phi}{a}\overline{u'v'}, \qquad \text{(A.6a)}\\
&\frac{\partial\overline{v}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{v}}{\partial\phi}+\overline{w}\frac{\partial\overline{v}}{\partial z^*}+f\overline{u}+\frac{\tan\phi}{a}(\overline{u})^2+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}-\overline{Y}\\
&\quad=-\frac{1}{a\cos\phi}\overline{u'\frac{\partial v'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}-\overline{w'\frac{\partial v'}{\partial z^*}}-\frac{\tan\phi}{a}\overline{u'^2}, \qquad \text{(A.6b)}\\
&\frac{\partial\overline{\Phi}}{\partial z^*}-\frac{Re^{-\kappa z^*/H}}{H}\overline{\theta}=0, \qquad \text{(A.6c)}\\
&\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\phi}(\overline{v}\cos\phi)\right]+\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w})=0, \qquad \text{(A.6d)}\\
&\frac{\partial\overline{\theta}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w}\frac{\partial\overline{\theta}}{\partial z^*}-\overline{Q}=-\frac{1}{a\cos\phi}\overline{u'\frac{\partial\theta'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial\theta'}{\partial\phi}}-\overline{w'\frac{\partial\theta'}{\partial z^*}} \qquad \text{(A.6e)}
\end{aligned}
$$

となる. ここで (A.5), (A.6) から東西平均からのずれに関する連続の式

$$
\frac{1}{a\cos\phi}\left[\frac{\partial u'}{\partial\lambda}+\frac{\partial}{\partial\phi}(v'\cos\phi)\right]+\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0 w')=0 \qquad \text{(A.7)}
$$

が得られる.

(A.7) を使って (A.6) を変形する. (A.7) に u' をかけてオイラー平均をとると

$$
\frac{1}{a\cos\phi}\overline{u'\frac{\partial u'}{\partial\lambda}}+\frac{1}{a}\overline{u'\frac{\partial v'}{\partial\phi}}-\frac{\tan\phi}{a}\overline{u'v'}+\overline{u'\frac{\partial w'}{\partial z^*}}+\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{u'w'}=0 \qquad \text{(A.8)}
$$

これを (A.6) に加えると

$$
\begin{aligned}
&\frac{\partial\overline{u}}{\partial t}+\frac{1}{a}\overline{v}\frac{\partial\overline{u}}{\partial\phi}+\overline{w}\frac{\partial\overline{u}}{\partial z^*}-f\overline{v}-\frac{\tan\phi}{a}\overline{uv}-\overline{X}\\
&\quad=-\frac{2}{a\cos\phi}\overline{u'\frac{\partial u'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial u'}{\partial\phi}}-\overline{w'\frac{\partial u'}{\partial z^*}}-\frac{1}{a}\overline{u'\frac{\partial v'}{\partial\phi}}+\frac{2\tan\phi}{a}\overline{u'v'}-\overline{u'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{u'w'}
\end{aligned}
$$

ここで

$$
\begin{aligned}
-\frac{2}{a\cos\phi}\overline{u'\frac{\partial u'}{\partial\lambda}}&=-\frac{1}{a\cos\phi}\overline{\frac{\partial(u')^2}{\partial\lambda}}=0,\\
-\frac{1}{a}\overline{v'\frac{\partial u'}{\partial\phi}}-\frac{1}{a}\overline{u'\frac{\partial v'}{\partial\phi}}+\frac{2\tan\phi}{a}\overline{u'v'}&=-\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}(\overline{v'u'}\cos^2\phi),\\
-\overline{w'\frac{\partial u'}{\partial z^*}}-\overline{u'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{u'w'}&=-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'u'})
\end{aligned}
$$

を用いると,

$$
\begin{aligned}
&\frac{\partial\overline{u}}{\partial t}+\frac{1}{a}\overline{v}\frac{\partial\overline{u}}{\partial\phi}+\overline{w}\frac{\partial\overline{u}}{\partial z^*}-f\overline{v}-\frac{\tan\phi}{a}\overline{u}\,\overline{v}-\overline{X}\\
&\quad=-\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}(\overline{v'u'}\cos^2\phi)-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'u'})
\end{aligned}
$$

と書くことができる. (A.6) に関しても同様に, (A.7) に v' をかけてオイラー平均をとった式

$$
\frac{1}{a\cos\phi}\overline{v'\frac{\partial u'}{\partial\lambda}}+\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}+\frac{\tan\phi}{a}\overline{v'^2}+\overline{v'\frac{\partial w'}{\partial z^*}}+\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{v'w'}=0 \tag{A.9}
$$

を (A.6) に加えると

$$
\begin{aligned}
&\frac{\partial\overline{v}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{v}}{\partial\phi}+\overline{w}\frac{\partial\overline{v}}{\partial z^*}+f\overline{u}+\frac{\tan\phi}{a}(\overline{u})^2+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}-\overline{Y}\\
&\quad=-\frac{1}{a\cos\phi}\overline{u'\frac{\partial v'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}-\overline{w'\frac{\partial v'}{\partial z^*}}-\frac{\tan\phi}{a}\overline{u'^2}\\
&\qquad-\frac{1}{a\cos\phi}\overline{v'\frac{\partial u'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}+\frac{\tan\phi}{a}\overline{v'^2}-\overline{v'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{v'w'}
\end{aligned}
$$

が得られる. ここで

$$
\begin{aligned}
-\frac{1}{a\cos\phi}\overline{u'\frac{\partial v'}{\partial\lambda}}-\frac{1}{a\cos\phi}\overline{v'\frac{\partial u'}{\partial\lambda}} &= -\frac{1}{a\cos\phi}\overline{\frac{\partial(u'v')}{\partial\lambda}}=0,\\
-\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}-\frac{1}{a}\overline{v'\frac{\partial v'}{\partial\phi}}+\frac{\tan\phi}{a}\overline{v'^2} &= -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\overline{v'^2}\right)\\
-\overline{w'\frac{\partial v'}{\partial z^*}}-\overline{v'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{v'w'} &= -\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{v'w'}\right)
\end{aligned} \tag{A.10}
$$

を用いると

$$
\begin{aligned}
&\frac{\partial\overline{v}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{v}}{\partial\phi}+\overline{w}\ DP\overline{v}z^*+f\overline{u}+\frac{\tan\phi}{a}(\overline{u})^2+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}-\overline{Y}\\
&\quad=-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\overline{v'^2}\right)-\frac{\tan\phi}{a}\overline{u'^2}-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{v'w'}\right)
\end{aligned}
$$

と書くことができる. (A.6) についても同様に, (A.7) に θ' をかけてオイラー平均をとった式

$$
\frac{1}{a\cos\phi}\overline{\theta'\frac{\partial u'}{\partial\lambda}}+\frac{1}{a}\overline{\theta'\frac{\partial v'}{\partial\phi}}-\frac{\tan\phi}{a}\overline{\theta'v'}+\overline{\theta'\frac{\partial w'}{\partial z^*}}+\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{\theta'w'}=0 \tag{A.11}
$$

を (A.6) に加えると

$$
\begin{aligned}
&\frac{\partial\overline{\theta}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w}\frac{\partial\overline{\theta}}{\partial z^*}-\overline{Q}\\
&\quad=-\frac{1}{a\cos\phi}\overline{u'\frac{\partial\theta'}{\partial\lambda}}-\frac{1}{a}\overline{v'\frac{\partial\theta'}{\partial\phi}}-\overline{w'\frac{\partial\theta'}{\partial z^*}}\\
&\qquad-\frac{1}{a\cos\phi}\overline{\theta'\frac{\partial u'}{\partial\lambda}}-\frac{1}{a}\overline{\theta'\frac{\partial v'}{\partial\phi}}+\frac{\tan\phi}{a}\overline{\theta'v'}-\overline{\theta'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{\theta'w'}
\end{aligned}
$$

が得られる. ここで

$$
\begin{aligned}
-\frac{1}{a\cos\phi}\overline{u'\frac{\partial\theta'}{\partial\lambda}}-\frac{1}{a\cos\phi}\overline{\theta'\frac{\partial u'}{\partial\lambda}} &= -\frac{1}{a\cos\phi}\overline{\frac{\partial(u'\theta')}{\partial\lambda}}=0,\\
-\frac{1}{a}\overline{v'\frac{\partial\theta'}{\partial\phi}}-\frac{1}{a}\overline{\theta'\frac{\partial v'}{\partial\phi}}+\frac{\tan\phi}{a}\overline{\theta'v'} &= -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\overline{v'\theta'}\right)\\
-\overline{w'\frac{\partial\theta'}{\partial z^*}}-\overline{\theta'\frac{\partial w'}{\partial z^*}}-\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial z^*}\overline{\theta'w'} &= -\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{w'\theta'}\right)
\end{aligned}
$$

を用いると

$$
\frac{\partial\overline{\theta}}{\partial t}+\frac{\overline{v}}{a}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w}\frac{\partial\overline{\theta}}{\partial z^*}-\overline{Q}=-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\overline{v'\theta'}\right)-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{w'\theta'}\right)
$$

となる.

以上をまとめると, 以下の**オイラー平均方程式**が得られる.

$$\frac{\partial \overline{u}}{\partial t} + \frac{1}{a}\overline{v}\frac{\partial \overline{u}}{\partial \phi} + \overline{w}\frac{\partial \overline{u}}{\partial z^*} - f\overline{v} - \frac{\tan\phi}{a}\overline{u}\ \overline{v} - \overline{X}$$
$$= -\frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}(\overline{v'u'}\cos^2\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'u'}), \tag{A.12a}$$

$$\frac{\partial \overline{v}}{\partial t} + \frac{\overline{v}}{a}\frac{\partial \overline{v}}{\partial \phi} + \overline{w}\frac{\partial \overline{v}}{\partial z^*} + f\overline{u} + \frac{\tan\phi}{a}(\overline{u})^2 + \frac{1}{a}\frac{\partial \overline{\Phi}}{\partial \phi} - \overline{Y}$$
$$= -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{v'^2}\cos\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{v'w'}) - \overline{u'^2}\frac{\tan\phi}{a}, \tag{A.12b}$$

$$\frac{\partial \overline{\Phi}}{\partial z^*} - \frac{Re^{-\kappa z^*/H}}{H}\overline{\theta} = 0, \tag{A.12c}$$

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{v}\cos\phi) + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w}) = 0, \tag{A.12d}$$

$$\frac{\partial \overline{\theta}}{\partial t} + \frac{\overline{v}}{a}\frac{\partial \overline{\theta}}{\partial \phi} + \overline{w}\frac{\partial \overline{\theta}}{\partial z^*} - \overline{Q} = -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{v'\theta'}\cos\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'}). \tag{A.12e}$$

1.3 変形オイラー平均方程式系

(A.12) を EP フラックス, 残差循環を用いて書き直す. EP フラックス, 残差循環は以下のように定義する.

$$\overline{v}^* = \overline{v} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right) \tag{A.13a}$$

$$\overline{w}^* = \overline{w} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right) \tag{A.13b}$$

$$\begin{aligned}
F_\phi &= \rho_0 a\cos\phi\left(\frac{\partial\overline{u}}{\partial\overline{z^*}}\frac{\overline{v'\theta'}}{\frac{\partial\bar{\theta}}{\partial z^*}} - \overline{u'v'}\right) \\
F_z^* &= \rho_0 a\cos\phi\left(\left[f - \frac{\frac{\partial\overline{u}\cos\phi}{\partial\phi}}{a\cos\phi}\right]\frac{\overline{v'\theta'}}{\frac{\partial\bar{\theta}}{\partial z^*}} - \overline{u'w'}\right)
\end{aligned}$$

まず連続の式を書き換える. (A.12) に (A.13), (A.13) を代入すると

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left[\left\{\overline{v}^* + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\}\cos\phi\right]$$
$$+\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left[\rho_0\left\{\overline{w}^* - \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\}\right] = 0,$$
$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\overline{v}^*\cos\phi\right) + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{w}^*\right)$$
$$+\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left\{\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\cos\phi\right\} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left\{\rho_0\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\} = 0.$$

この第三項と第四項だけを取り出すと

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left\{\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\cos\phi\right\} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left\{\rho_0\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\}$$
$$= \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\phi}\left\{\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\cos\phi\right\} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left\{\rho_0\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\}\right]$$
$$= \frac{1}{a\cos\phi}\left[\frac{1}{\rho_0}\frac{\partial}{\partial\phi}\left\{\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\cos\phi\right)\right\} - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left\{\frac{\partial}{\partial\phi}\left(\rho_0\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^*}}\right)\right\}\right]$$
$$= 0.$$

したがって, 連続の式は以下のようになる.

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\overline{v}^*\cos\phi\right) + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\overline{w}^*\right) = 0. \tag{A.14}$$

次に u の式を書き換える. (A.12) に (A.13), (A.13) を代入すると

$$
\begin{aligned}
&\frac{\partial \overline{u}}{\partial t} + \frac{1}{a}\left[\overline{v}^* + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\right]\frac{\partial \overline{u}}{\partial \phi} + \left[\overline{w}^* - \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\right]\frac{\partial \overline{u}}{\partial z^*} \\
&\qquad - f\left[\overline{v}^* + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\right] - \frac{\tan\phi}{a}\overline{u}\left[\overline{v}^* + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\right] - \overline{X} \\
&\quad = -\frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}(\overline{v'u'}\cos^2\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0 \overline{w'u'}), \\
&\frac{\partial \overline{u}}{\partial t} + \frac{\overline{v}^*}{a}\frac{\partial \overline{u}}{\partial \phi} + \overline{w}^*\frac{\partial \overline{u}}{\partial z^*} - f\overline{v}^* - \frac{\tan\phi}{a}\overline{u}\ \overline{v}^* - \overline{X} \\
&\quad = -\frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}(\overline{v'u'}\cos^2\phi) + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\frac{\partial \overline{u}}{\partial z^*} \\
&\qquad + f\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0 \overline{w'u'}) \\
&\qquad - \frac{1}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\frac{\partial \overline{u}}{\partial \phi} + \frac{\tan\phi}{a}\overline{u}\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right), \\
&\frac{\partial \overline{u}}{\partial t} + \frac{\overline{v}^*}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{u}\cos\phi) + \overline{w}^*\frac{\partial \overline{u}}{\partial z^*} - f\overline{v}^* - \overline{X} \\
&\quad = -\frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial \phi}(\rho_0 a\overline{v'u'}\cos^2\phi) + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\frac{\partial \overline{u}}{\partial z^*} \\
&\qquad + \frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left(f\rho_0 a\cos\phi \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right) - \frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}(\rho_0 a\cos\phi\overline{w'u'}) \\
&\qquad - \frac{1}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right)\frac{\partial \overline{u}}{\partial \phi} + \frac{\tan\phi}{a}\overline{u}\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 \frac{\overline{v'\theta'}}{\frac{\partial \theta}{\partial z^*}}\right) \qquad \text{(A.15)}
\end{aligned}
$$

(1.3) の右辺を以下のように変形する.

$$
\begin{aligned}
&-\frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)+\frac{1}{\rho_0 a^2\cos^2\phi}\rho_0 a\cos\phi\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\\
&\qquad+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left(f\rho_0 a\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)-\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}(\rho_0 a\cos\phi\overline{w'u'})\\
&\qquad-\frac{1}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}\right)+\frac{1}{\rho_0 a}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\left(\frac{\partial\overline{u}}{\partial\phi}\right)\\
&\qquad+\frac{\tan\phi}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\overline{u}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)-\frac{\tan\phi}{\rho_0 a}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}(\overline{u})\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)+\rho_0 a\cos\phi\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\\
&\quad+\frac{1}{\rho_0 a}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\left(\frac{\partial\overline{u}}{\partial\phi}\right)-\frac{\tan\phi}{\rho_0 a}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\\
&\quad+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\left(f\rho_0 a\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)-\rho_0 a\cos\phi\overline{w'u'}\right]\\
&\quad-\frac{1}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}\right)+\frac{\tan\phi}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\overline{u}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)+\rho_0 a\cos\phi\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\\
&\quad+\frac{1}{\rho_0 a^2\cos^2\phi}\left[\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\left(\frac{\partial\overline{u}}{\partial\phi}\right)-\rho_0 a\cos^2\phi\tan\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\right]\\
&\quad+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\left(f\rho_0 a\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)-\rho_0 a\cos\phi\overline{w'u'}\right]\\
&\quad+\frac{1}{\rho_0 a\cos\phi}\left[-\cos\phi\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}\right)+\cos\phi\tan\phi\frac{\partial}{\partial z^*}\left(\overline{u}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)+\rho_0 a\cos\phi\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\\
&\quad+\frac{1}{\rho_0 a^2\cos^2\phi}\left[\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{u}}{\partial z^*}\right)+\cos\phi\frac{\partial}{\partial\phi}\left(\rho_0 a\cos\phi\right)\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\right]\\
&\quad+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[f\rho_0 a\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}-\rho_0 a\cos\phi\overline{w'u'}\right]\\
&\quad+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[-\rho_0\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}+\sin\phi\overline{u}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right]
\end{aligned}
\tag{A.16}
$$

(1.3) の第一項と第二項だけ取り出すと

$$
\begin{aligned}
&\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)+\rho_0 a\cos\phi\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\right)\right]\\
&\quad+\frac{1}{\rho_0 a^2\cos^2\phi}\left[\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{u}}{\partial z^*}\right)+\cos\phi\frac{\partial}{\partial\phi}\left(\rho_0 a\cos\phi\right)\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)\right]\\
&\quad+\frac{1}{\rho_0 a^2\cos^2\phi}\left[\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{u}}{\partial z^*}\right)+\frac{\partial\overline{u}}{\partial z^*}\frac{\partial}{\partial\phi}\left(\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\right)\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\left[-\frac{\partial}{\partial\phi}(\rho_0 a\overline{v'u'}\cos^2\phi)\right]+\frac{1}{\rho_0 a^2\cos^2\phi}\left[\frac{\partial}{\partial\phi}\left(\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\right)\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial\phi}\left[-\rho_0 a\overline{v'u'}\cos^2\phi+\rho_0 a\cos^2\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial\overline{u}}{\partial z^*}\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial\phi}\left[\rho_0 a\cos^2\phi\left\{\frac{\partial\overline{u}}{\partial z^*}\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\overline{v'u'}\right\}\right]\\
&=\frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos\phi F_\phi^*\right)
\end{aligned}
$$

(1.3) の第三項と第四項だけ取り出すと

$$
\begin{aligned}
&\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[f\rho_0 a\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\rho_0 a\cos\phi\overline{w'u'}\right]+\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[-\rho_0\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}+\sin\phi\overline{u}\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}\right]\\
&=\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\rho_0 a\cos\phi\left\{f\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\overline{w'u'}-\frac{\overline{v'\theta'}}{a\frac{\partial\theta}{\partial z^*}}\frac{\partial\overline{u}}{\partial\phi}+\sin\phi\overline{u}\frac{\overline{v'\theta'}}{a\cos\phi\frac{\partial\theta}{\partial z^*}}\right\}\right]\\
&=\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\rho_0 a\cos\phi\left\{f\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\left(\cos\phi\frac{\partial\overline{u}}{\partial\phi}-\sin\phi\overline{u}\right)\frac{\overline{v'\theta'}}{a\cos\phi\frac{\partial\theta}{\partial z^*}}-\overline{w'u'}\right\}\right]\\
&=\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\rho_0 a\cos\phi\left\{f\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\frac{\partial(\overline{u}\cos\phi)}{\partial\phi}\frac{\overline{v'\theta'}}{a\cos\phi\frac{\partial\theta}{\partial z^*}}-\overline{w'u'}\right\}\right]\\
&=\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial z^*}\left[\rho_0 a\cos\phi\left\{\left(f-\frac{\frac{\partial(\overline{u}\cos\phi)}{\partial\phi}}{a\cos\phi}\right)\frac{\overline{v'\theta'}}{\frac{\partial\theta}{\partial z^*}}-\overline{w'u'}\right\}\right]\\
&=\frac{1}{\rho_0 a\cos\phi}\frac{\partial F_z^*}{\partial z^*}
\end{aligned}
$$

以上より, (1.3) は次のようになる.

$$\frac{\partial \overline{u}}{\partial t} + \frac{\overline{v}^*}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\overline{u}\cos\phi\right) + \overline{w}^*\frac{\partial \overline{u}}{\partial z^*} - f\overline{v}^* - \overline{X} = \frac{1}{\rho_0 a^2\cos^2\phi}\frac{\partial}{\partial \phi}\left(\cos\phi F_\phi^*\right) + \frac{1}{\rho_0 a\cos\phi}\frac{\partial F_z^*}{\partial z^*},$$

$$\frac{\partial \overline{u}}{\partial t} + \frac{\overline{v}^*}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\overline{u}\cos\phi\right) + \overline{w}^*\frac{\partial \overline{u}}{\partial z^*} - f\overline{v}^* - \overline{X} = \frac{1}{\rho_0 a\cos\phi}\nabla\cdot\boldsymbol{F}.$$

ここで, 子午面内の発散を以下のように表した.

$$\nabla\cdot\boldsymbol{F} = \frac{1}{a\cos\phi}\frac{\partial(\cos\phi F_\phi)}{\partial \phi} + \frac{\partial F_{z^*}}{\partial z^*} \tag{A.17}$$

次に熱力学の式を書き換える. (A.12) に (A.13), (A.13) を代入すると

$$\begin{aligned}
&\frac{\partial \overline{\theta}}{\partial t} + \frac{1}{a}\left[\overline{v}^* + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\frac{\partial \overline{\theta}}{\partial \phi} + \left[\overline{w}^* - \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\right]\frac{\partial \overline{\theta}}{\partial z^*} - \overline{Q} \\
&\quad = -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{v'\theta'}\cos\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'}), \\
&\frac{\partial \overline{\theta}}{\partial t} + \frac{\overline{v}^*}{a}\frac{\partial \overline{\theta}}{\partial \phi} + \overline{w}^*\frac{\partial \overline{\theta}}{\partial z^*} - \overline{Q} \\
&\quad = -\frac{1}{\rho_0 a}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\frac{\partial \overline{\theta}}{\partial \phi} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\frac{\partial \overline{\theta}}{\partial z^*} \\
&\qquad - \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\overline{v'\theta'}\cos\phi) - \frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'})
\end{aligned}$$

となる. この右辺を更に変形すると

$$
\begin{aligned}
&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\right)\frac{\partial\overline{\theta}}{\partial\phi}+\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\frac{\partial\overline{\theta}}{\partial z^*}\\
&\quad-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{v'\theta'}\cos\phi)-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'})\\
=&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}\right)+\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\frac{\partial\overline{\theta}}{\partial\phi}\\
&\quad+\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\phi}\left(\cos\phi\overline{v'\theta'}\right)\frac{1}{\frac{\partial\overline{\theta}}{\partial z^*}}+\cos\phi\overline{v'\theta'}\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{\theta}}{\partial z^*}\right)^{-1}\right]\frac{\partial\overline{\theta}}{\partial z^*}\\
&\quad-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{v'\theta'}\cos\phi)-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'})\\
=&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}\right)+\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\frac{\partial\overline{\theta}}{\partial\phi}+\frac{1}{a}\overline{v'\theta'}\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{\theta}}{\partial z^*}\right)^{-1}\frac{\partial\overline{\theta}}{\partial z^*}-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}(\rho_0\overline{w'\theta'})\\
=&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left[\rho_0\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}+\rho_0\overline{w'\theta'}\right]+\frac{\overline{v'\theta'}}{a}\left[\frac{1}{\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial}{\partial z^*}\frac{\partial\overline{\theta}}{\partial\phi}+\frac{\partial}{\partial\phi}\left(\frac{\partial\overline{\theta}}{\partial z^*}\right)^{-1}\frac{\partial\overline{\theta}}{\partial z^*}\right]\\
=&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left[\rho_0\left(\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w'\theta'}\right)\right]+\frac{\overline{v'\theta'}}{a}\frac{\partial}{\partial\phi}\left(\frac{\frac{\partial\overline{\theta}}{\partial z^*}}{\frac{\partial\overline{\theta}}{\partial z^*}}\right)\\
=&-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left[\rho_0\left(\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w'\theta'}\right)\right].
\end{aligned}
$$

これより, 熱力学の式は以下のようになる.

$$
\frac{\partial\overline{\theta}}{\partial t}+\frac{\overline{v}^*}{a}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w}^*\frac{\partial\overline{\theta}}{\partial z^*}-\overline{Q}=-\frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left[\rho_0\left(\frac{\overline{v'\theta'}}{a\frac{\partial\overline{\theta}}{\partial z^*}}\frac{\partial\overline{\theta}}{\partial\phi}+\overline{w'\theta'}\right)\right].
$$

最後に v の式について考える. (A.12) に (A.13), (A.13) を代入すると

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]+\frac{1}{a}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\frac{\partial}{\partial\phi}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\\
&\qquad+\left[\overline{w}^{*}-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\frac{\partial}{\partial z^{*}}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\\
&\qquad+f\overline{u}+\frac{\tan\phi}{a}(\overline{u})^2+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}-\overline{Y}\\
&\quad=-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{v'^2}\cos\phi)-\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}(\rho_0\overline{v'w'})-\overline{u'^2}\frac{\tan\phi}{a},
\end{aligned}
$$

$$
\begin{aligned}
&f\overline{u}+\frac{\tan\phi}{a}(\overline{u})^2+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}\\
&\quad=-\frac{\partial}{\partial t}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]-\frac{1}{a}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\frac{\partial}{\partial\phi}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\\
&\qquad-\left[\overline{w}^{*}-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\frac{\partial}{\partial z^{*}}\left[\overline{v}^{*}+\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}\left(\rho_0\frac{\overline{v'\theta'}}{\frac{\overline{\partial\theta}}{\partial z^{*}}}\right)\right]\\
&\qquad-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{v'^2}\cos\phi)-\frac{1}{\rho_0}\frac{\partial}{\partial z^{*}}(\rho_0\overline{v'w'})-\overline{u'^2}\frac{\tan\phi}{a}+\overline{Y}
\end{aligned}
$$

Andrews *et al.* (1987) によれば, この式の右辺の量は左辺に比べれば小さい. 右辺の項を全てまとめて G と書くと v の式は次のようになる.

$$
\overline{u}\left(f+\frac{\tan\phi}{a}\overline{u}\right)+\frac{1}{a}\frac{\partial\overline{\Phi}}{\partial\phi}=G.
$$

以上をまとめると, 以下の**変形オイラー平均方程式**が得られる.

$$\frac{\partial \overline{u}}{\partial t} + \overline{v}^* \left[\frac{1}{a\cos\phi} \frac{\partial}{\partial \phi} (\overline{u}\cos\phi) - f \right] + \overline{w}^* \frac{\partial \overline{u}}{\partial z^*} - \overline{X} = \frac{1}{\rho_0 a \cos\phi} \nabla \cdot \boldsymbol{F}, \quad \text{(A.18a)}$$

$$\overline{u} \left(f + \overline{u} \frac{\tan\phi}{a} \right) + \frac{1}{a} \frac{\partial \overline{\Phi}}{\partial \phi} = G. \quad \text{(A.18b)}$$

$$\frac{\partial \overline{\Phi}}{\partial z^*} - \frac{R e^{-\kappa z^*/H}}{H} \overline{\theta} = 0. \quad \text{(A.18c)}$$

$$\frac{1}{a\cos\phi} \left[\frac{\partial}{\partial \phi} (\overline{v}^* \cos\phi) \right] + \frac{1}{\rho_0} \frac{\partial}{\partial z^*} (\rho_0 \overline{w}^*) = 0. \quad \text{(A.18d)}$$

$$\frac{\partial \overline{\theta}}{\partial t} + \frac{\overline{v}^*}{a} \frac{\partial \overline{\theta}}{\partial \phi} + \overline{w}^* \frac{\partial \overline{\theta}}{\partial z^*} - \overline{Q} = -\frac{1}{\rho_0} \frac{\partial}{\partial z^*} \left[\rho_0 \left(\overline{v'\theta'} \frac{\frac{\partial \overline{\theta}}{\partial \phi}}{a \frac{\partial \overline{\theta}}{\partial z^*}} + \overline{w'\theta'} \right) \right]. \quad \text{(A.18e)}$$

関連図書

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