Magnetic Rossby waves in the Earth's core

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Waves in the Earth’s fluid core

Waves provide us with information about the ‘invisible’ system

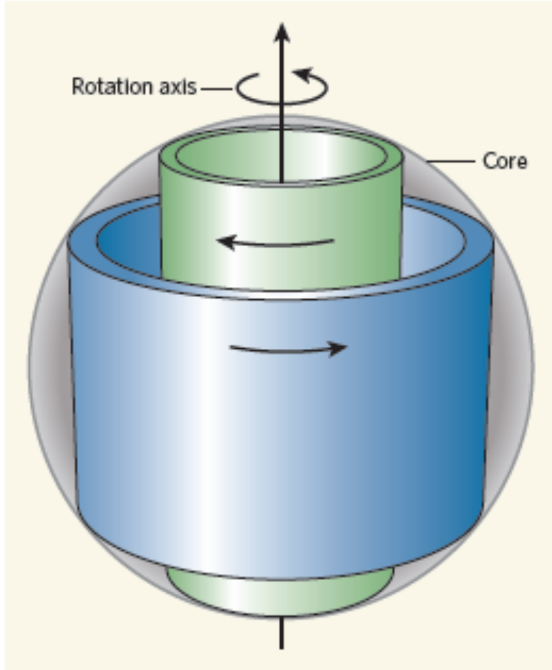
- **torsional Alfven waves** (e.g. Braginsky 1967, Zatman & Bloxham 1997)
  - axisymmetric, travelling in radius s
  - ~ 6 yrs traveltime: Bs >~ 2 mT (Gillet et al. 2010, 2015)
- axisymmetric MAC oscillations (e.g. Braginsky 1993)
  - in a thin, stably stratified layer at the top of the core?
  - ~ 60 yrs geomagnetic variation: H ~ 140 km? (Buffett 2014)
- slow magnetic Rossby waves (e.g. Hide 1966, Acheson 1978)
  - nonaxisymmetric, travelling in azimuth ϕ
  - ~ 300 yrs westward drift: $B\phi$ ~ 10 mT? (Hori et al. 2015)
- (fast magnetic) Rossby waves in a thin stable layer (e.g. Braginsky 1984)
  - ~ 6 yrs westward drift? (Chulliat et al. 2015)
  - in the solar tachocline also?: ~ 2 yrs westward? (McIntosh et al. 2017)

An axisymmetric mode: torsional Alfven waves

- A special class of Alfven waves
  (Braginsky 1970; also Roberts & Aurnou 2012)
  - the azimuthal momentum eq on cylindrical surfaces in the magnetostrophic balance gives a steady state (Taylor 1963)
  - cylindrical perturbations on the state

$$\frac{\partial^2}{\partial t^2}\frac{\langle\overline{u'_\phi}\rangle}{s} = \frac{1}{s^3 h\langle\overline{\rho}\rangle}\frac{\partial}{\partial s}\left(s^3 h\langle\overline{\rho}\rangle U_{\mathrm{A}}^2 \frac{\partial}{\partial s}\frac{\langle\overline{u'_\phi}\rangle}{s}\right)$$

» travel in radius s with the the z-mean Alfven speed $U_A = (\langle B_s^2\rangle/\langle\rho\rangle\mu_0)^{1/2}$



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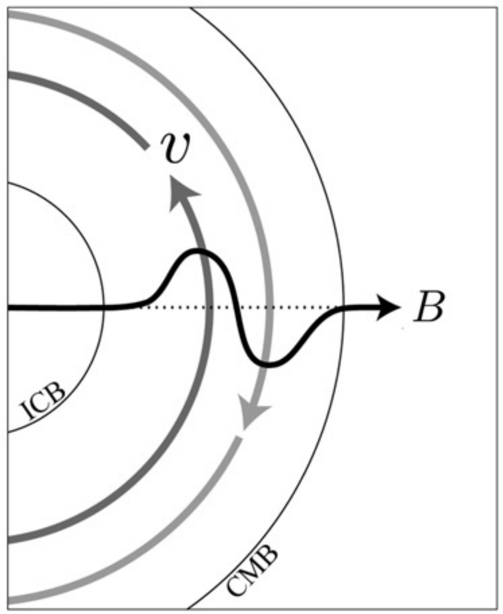

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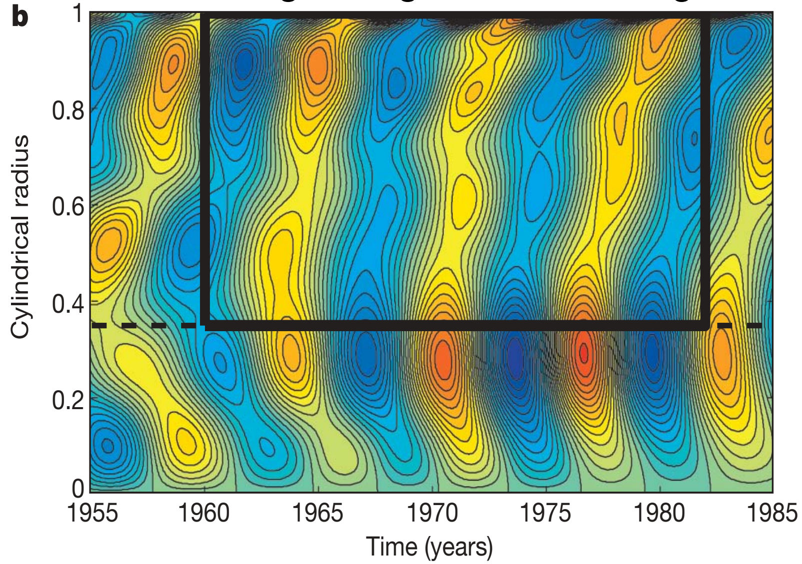
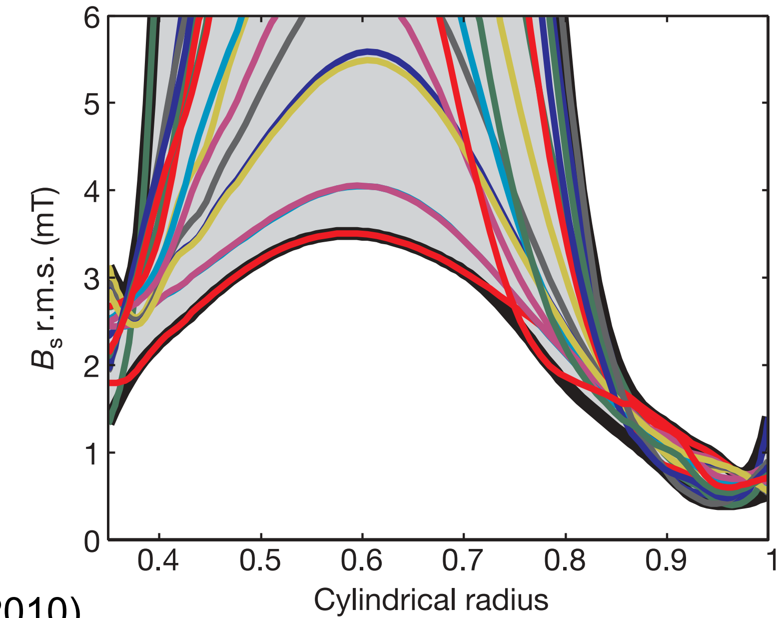
    - » travel in radius s with the the z-mean Alfven speed $U_A = (\langle B_s^2\rangle / \langle\rho\rangle \mu_0)^{1/2}$

- Data:
  - probably responsible for 6-7 year variations
    - » can account for the 6 year LOD change
  - the observed wave speed is used to infer the field strength within the core
    - » $\langle Bs^2\rangle^{1/2} \geq 2$ mT
    - » better fits with the scaling law

U_ϕ in a core flow model inverted from the geomagnetic variation gufm1

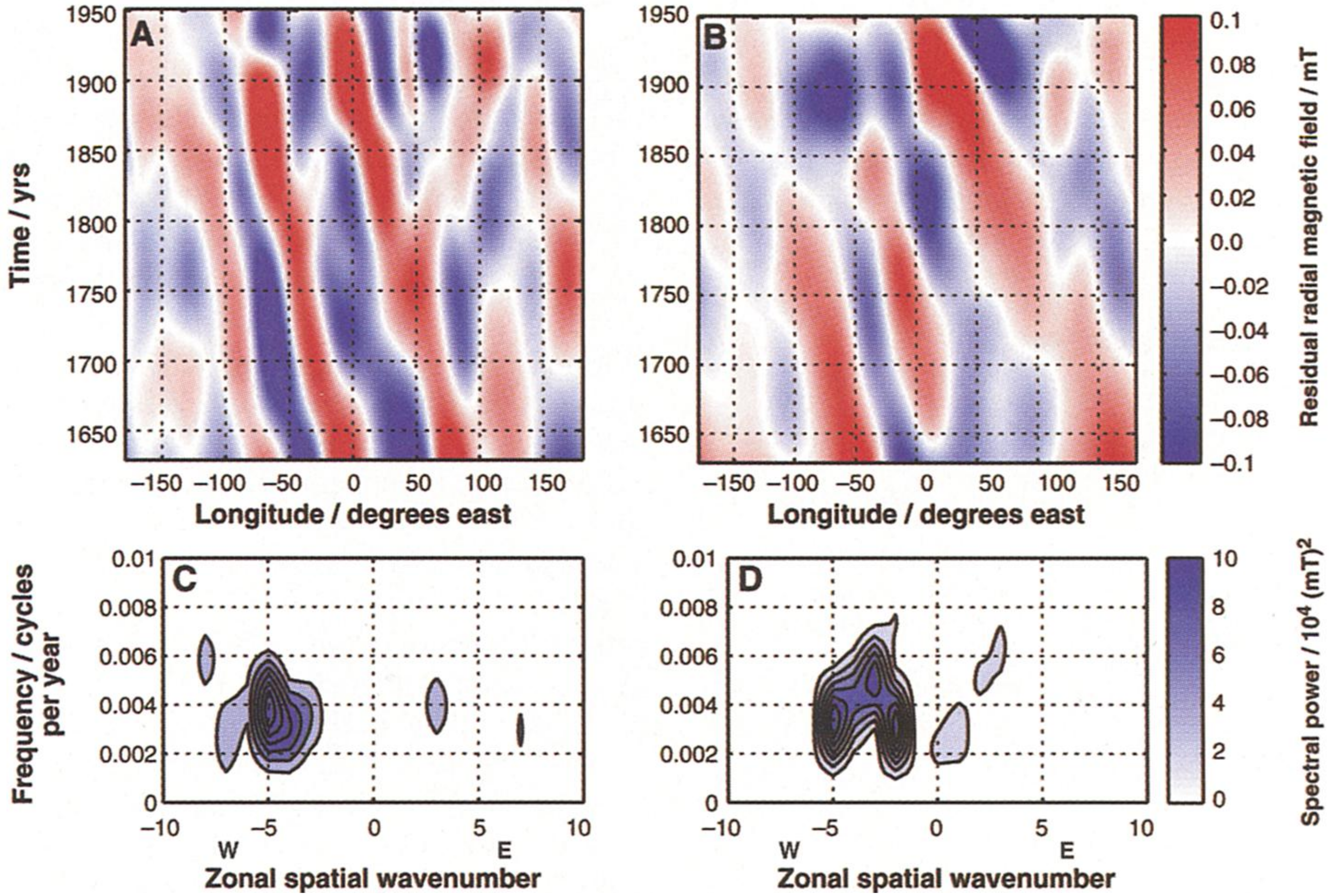




(Gillet et al. 2010)

Nonaxisymmetric waves in the core?

- Possibly related to the geomagnetic westward drift
  - the nonaxisymmetric part of the field moving in azimuth
    - significant in the Atlantic hemisphere: period ~ $3*10^2$ yrs
  - probably a mixture of flow advection (Bullard+ 1950) and wave propagation (Hide 1966)
    - → How can we separate the signal due to waves?



Nonaxisymmetric part of Br
at the surface of the core
at the equator / 40° S
(gufm1: Finlay & Jackson 2003)

Magnetic Rossby waves

- Key ingredients (Hide 1966; Acheson 1978; also Hori et al. 2015):
  - axial vorticity equation in a quasi-magnetostrophic balance (Λ=O(1); Ro, E<<1)

$$\rho \frac{\partial \xi_z'}{\partial t} - 2\rho\Omega \frac{\partial u_z'}{\partial z} = \hat{\boldsymbol{e}}_z \cdot \nabla \times (\boldsymbol{j}' \times \widetilde{\boldsymbol{B}})$$

  coupled with the induction equation

$$\frac{\partial \boldsymbol{b}'}{\partial t} = \widetilde{\boldsymbol{B}} \cdot \nabla \boldsymbol{u}'$$

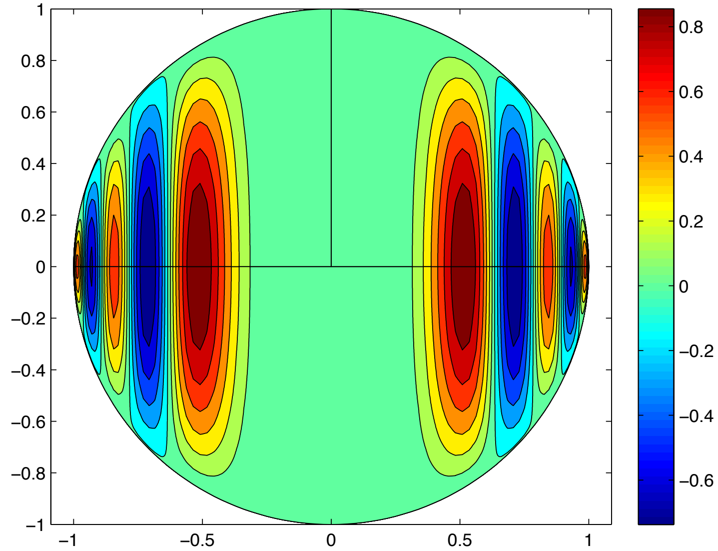
  - spherical geometry (topographic β–effect)
  - almost independent of z (quasi-geostrophic)
  - azimuthal length scales shorter than radial ones

- Dispersion relations about a mean flow: with a form of $e^{i(m\phi-\omega t)}$

$$\hat{\omega}_{\pm} = \hat{\omega}_R \left[\frac{1}{2} \pm \frac{1}{2} \sqrt{1 + 4\frac{\hat{\omega}_M^2}{\hat{\omega}_R^2}} \right]$$

where Rossby and Alfven frequencies

$$\hat{\omega}_R = \frac{2\Omega s^2}{(r_o^2 - s^2)m} \qquad \hat{\omega}_M^2 = \frac{m^2}{\rho\mu_0} \frac{\overline{\langle \widetilde{B_\phi^2} \rangle}}{s^2}$$



a QG eigenfunction for $B_\phi = B_0\, s\, e_\phi$ in a meridional section (after Malkus 1967)

Magnetic Rossby waves (cont'd)

- Fast modes:
  - $\omega_+ \rightarrow +\omega_R (1+\omega_M^2/\omega_R^2)$ in the limit $\omega_M^2/\omega_R^2 << 1$
  - essentially (nonmag) Rossby waves (Busse 1986)
  - travelling progradely (eastward) with timescales of O(months) in the fluid core

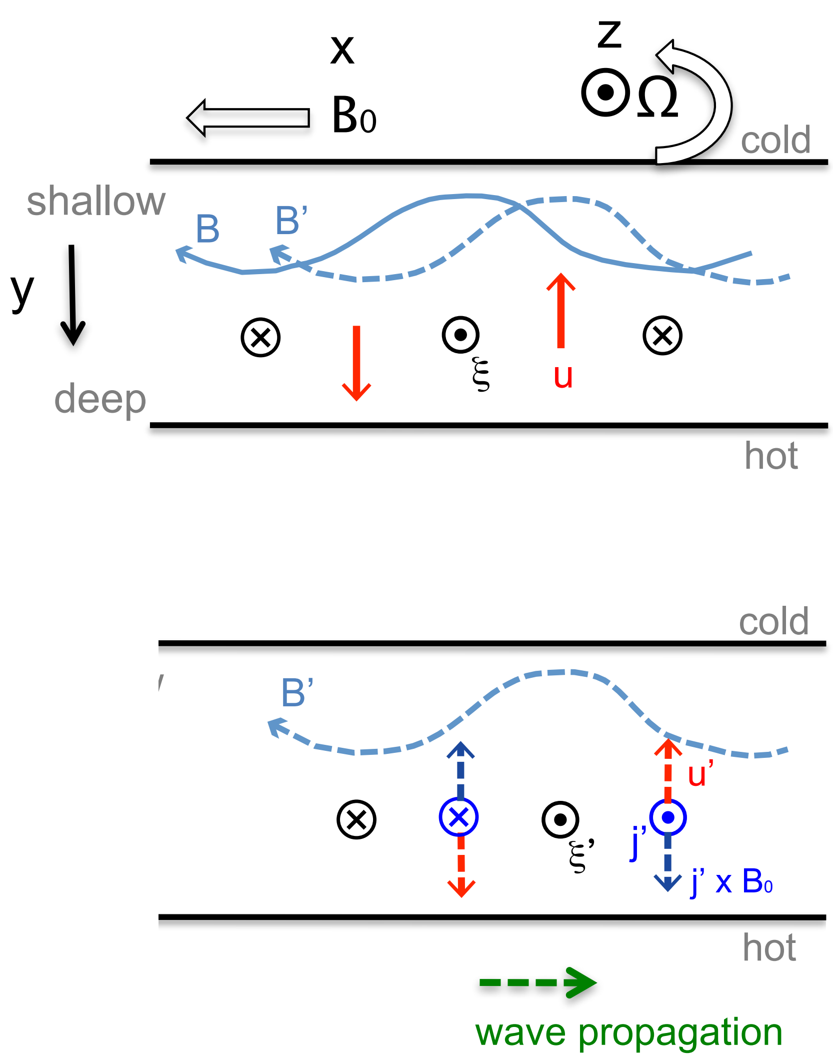
- Slow modes:
  - $\omega_- \rightarrow -\omega_M^2/\omega_R$ in the limit $\omega_M^2/\omega_R^2 << 1$

$$\hat{\omega}_{MR} = -\frac{\hat{\omega}_M^2}{\hat{\omega}_R} = -\frac{m^3(r_o^2 - s^2)\langle\widetilde{\overline{B_\phi^2}}\rangle}{2\rho\mu_0\Omega s^4}$$

  - travelling retrogradely (westward) along the toroidal field B_ϕ on timescales of O(10^2 years)
    - cf. torsional Alfven waves along Bs
  - highly dispersive
  - the governing equations (Cartesian)

$$\frac{\partial j_z'}{\partial t} = \frac{B_{0x}}{\mu_0}\frac{\partial \xi_z'}{\partial x}$$

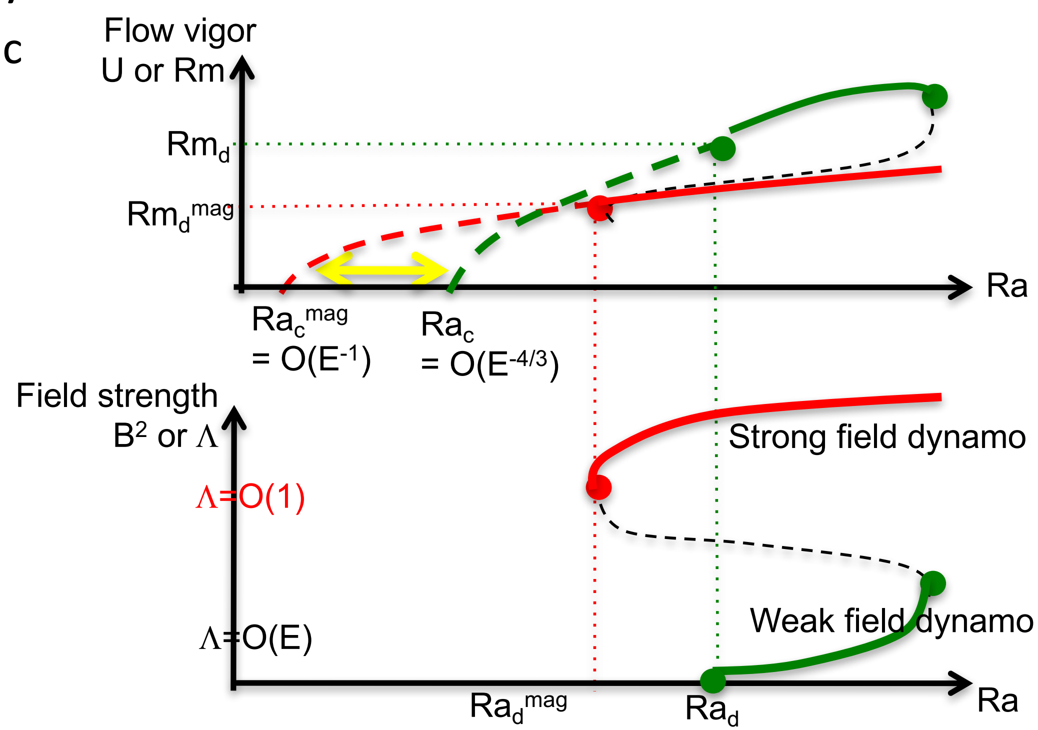
$$-\frac{4\rho\Omega\chi}{L}u_y' = B_{0x}\frac{\partial j_z'}{\partial x}$$



(Hori, Takehiro & Shimizu, 2014)

Waves hint at strong-field dynamos?

- Linear, rotating magnetoconvection
  (e.g. Chandrasekahr 1961, Fearn 1979; also Zhang & Schubert 2000):
  - as magnetic field is strengthened to Λ=O(1), the thermal stability Ra_{crit}, the preferred wavenumber k_{crit}, and wave frequency ω_{crit} drop
  - dynamos hypothesized in the regime:
    'strong-field' dynamos (e.g. Roberts 1978)
  - Note: all three effects not necessarily
    - depend on the background magnetic field, boundary conditions, etc.



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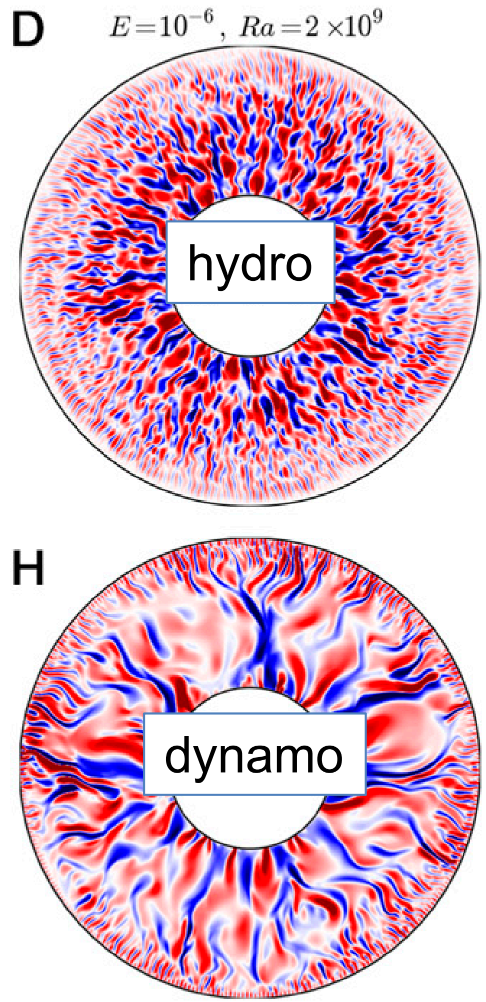
  - Note: all three effects not necessarily
    - depend on the background magnetic field, boundary conditions, etc.
- Convection-driven spherical dynamos likely approaching the regime (e.g. Yadav et al. 2016; Dormy 2016)
  - force balances
  - **flow properties?** (vigor/heat transfer/subcriticality, azimuthal length scales, and wave time scales)
    - cf. plane layer models



Radial velocity in the equatorial plane at E = 10^{-6}, Ra/Ra_c = 10, Pm/Pr = 0.5 (Yadav et al. 2016)

Convection-driven, spherical dynamo simulations

- Greatly studied for the past decades (e.g. Glatzmaier & Roberts 1995; Kageyama & Sato 1995; also reviews by Christensen & Wicht 2007; Jones 2011)
  – successful for reproducing observed features of planetary magnetic fields
  – a tool for understanding the dynamics with self-generated magnetic fields

- MHD dynamos driven by Boussinesq convection in rotating spherical shells:
  – Governing equations (dimensionless)

$$\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} &= \frac{Pm}{E}\left[2\hat{\boldsymbol{e}}_z \times \boldsymbol{u} - \nabla p + (\nabla\times\boldsymbol{B})\times\boldsymbol{B}\right] + \frac{Pm^2 Ra}{Pr} T\hat{\boldsymbol{e}}_r + Pm\nabla^2\boldsymbol{u} \\
\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T &= \frac{Pm}{Pr}\nabla^2 T - 1 \\
\frac{\partial B}{\partial t} &= \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) + \nabla^2\boldsymbol{B} \\
\nabla\cdot\boldsymbol{u} &= 0\,, \quad \nabla\cdot\boldsymbol{B} = 0
\end{aligned}$$

  – Parameters: modified Rayleigh, Ekman, kinetic/magnetic Prandtl numbers

$$Ra = \frac{g\alpha|\epsilon|D^5}{\nu\kappa\eta}\,, \quad E = \frac{\nu}{\Omega D^2}\,, \quad Pr = \frac{\nu}{\kappa}\,, \quad Pm = \frac{\nu}{\eta}$$

~ 16 Ra_{crit} = 10^{-4} - 10^{-6} = 1 = 1-5

  – Leeds spherical dynamo code: based on pseudo spectral method (e.g. Jones et al. 2011)

Slow MR waves in dynamo simulations

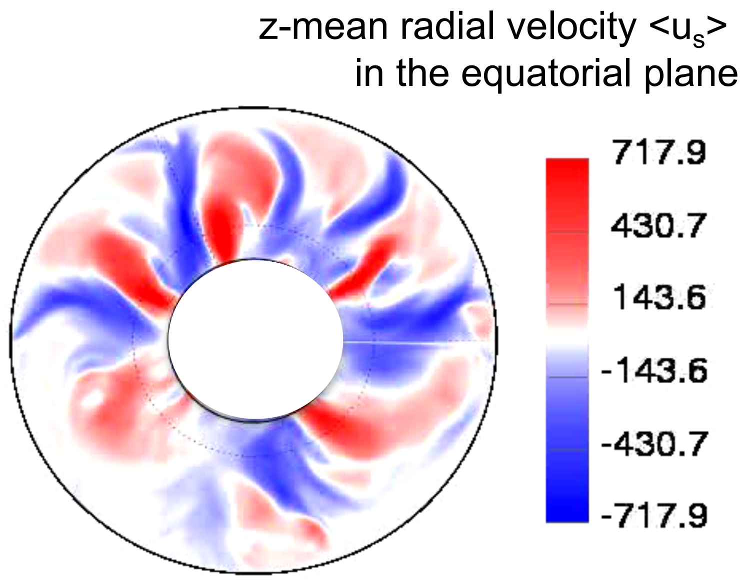
- Slow modes identified:
  - retrograde drifts commonly seen in dynamo simulations
  - their speeds accounted for by total phase speeds of wave and mean flow advection, $(\omega_{MR} + \omega_{adv})/m$, where

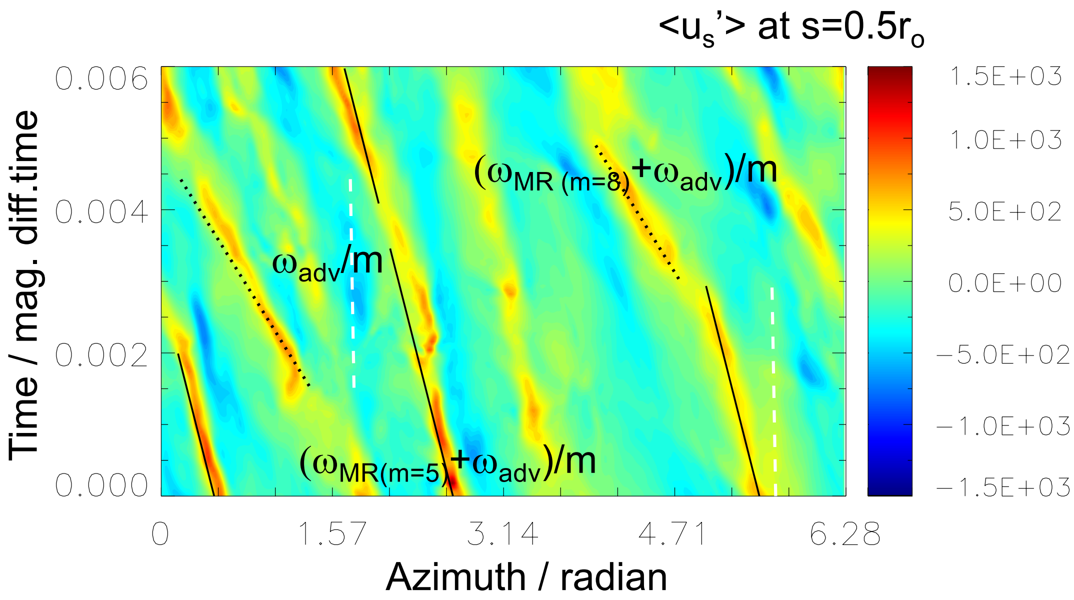
$$\hat{\omega}_{MR} = -\frac{\hat{\omega}_M^2}{\hat{\omega}_R} = -\frac{m^3(r_o^2 - s^2)\langle\widetilde{\overline{B_\phi^2}}\rangle}{2\rho\mu_0\Omega s^4}$$

$$\omega_{\mathrm{adv}} = \tilde{\zeta} m = \frac{\langle\widetilde{\overline{U_\phi}}\rangle}{s}\, m$$

  - 2D spectral analysis is crucial to distinguish each component

- Note: wave contribution depends on the radius s
  - wave ~< advection at larger s





at $E = 10^{-5}$, Pm/Pr = 5, $Ra/Ra_c = 8$ & $\Lambda \sim 22$
(Hori, Jones & Teed, 2015)

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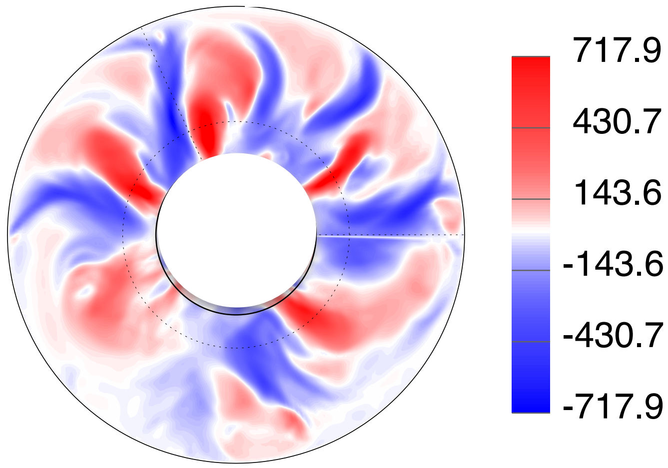
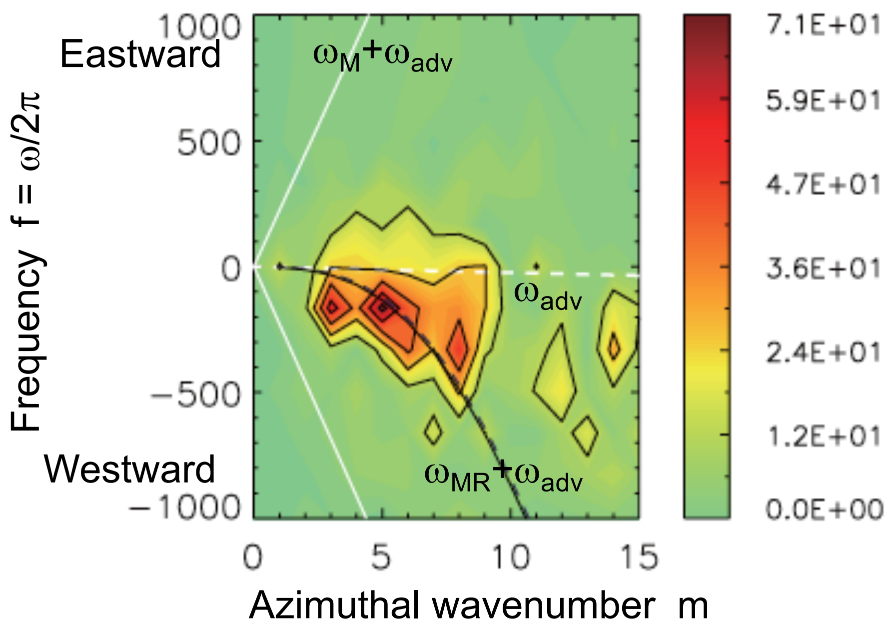


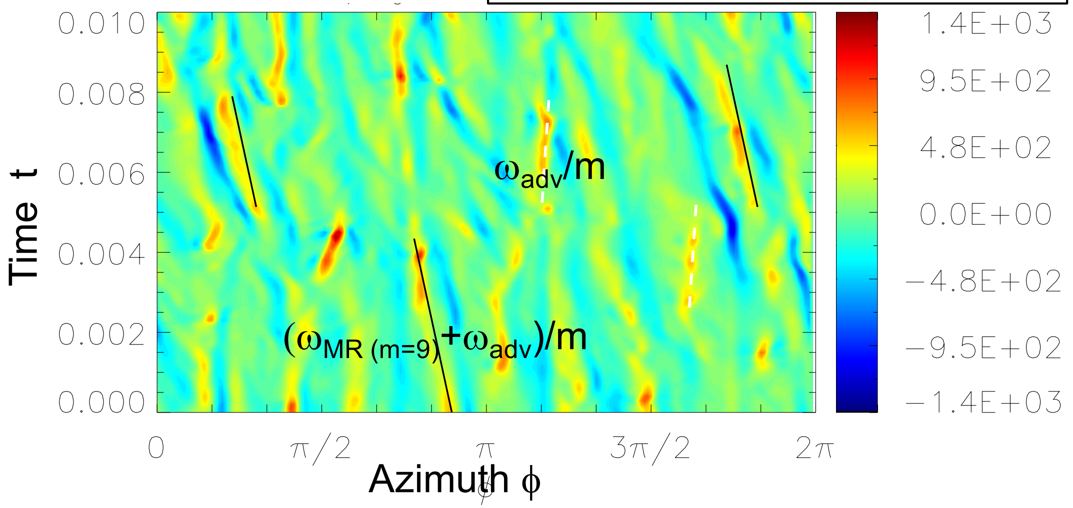


Exploring more cases

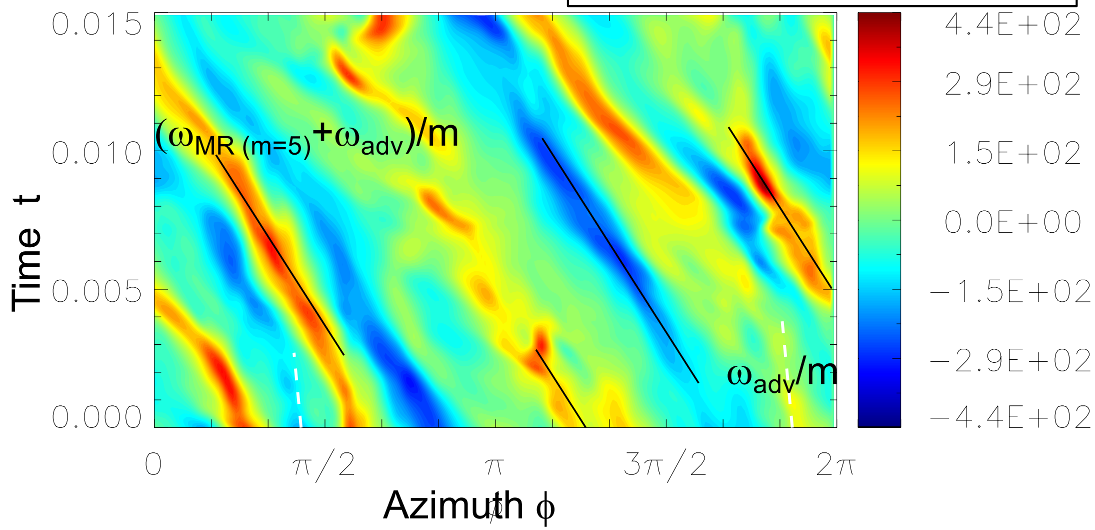
- MR waves were found in models when torsional waves were found
  - generated magnetic fields of non-reversing dipole
  - **for strong-field solutions (Λ >~ 2; Pm >= 5 or E =< 10^{-5}) ,** good Taylorization (< 0.2), good geostrophy (U'_C > 0.4)

- Note: excited azimuthal wave-numbers m vary
  - chosen by the convective instability
    - dependent on E, Ra, Λ, etc

E=$5*10^{-6}$, Pm=2, Ra/Ra_c = 16 & Λ ~ 6



E=10^{-4}, Pm=5, Ra/Ra_c = 8 & Λ ~ 22

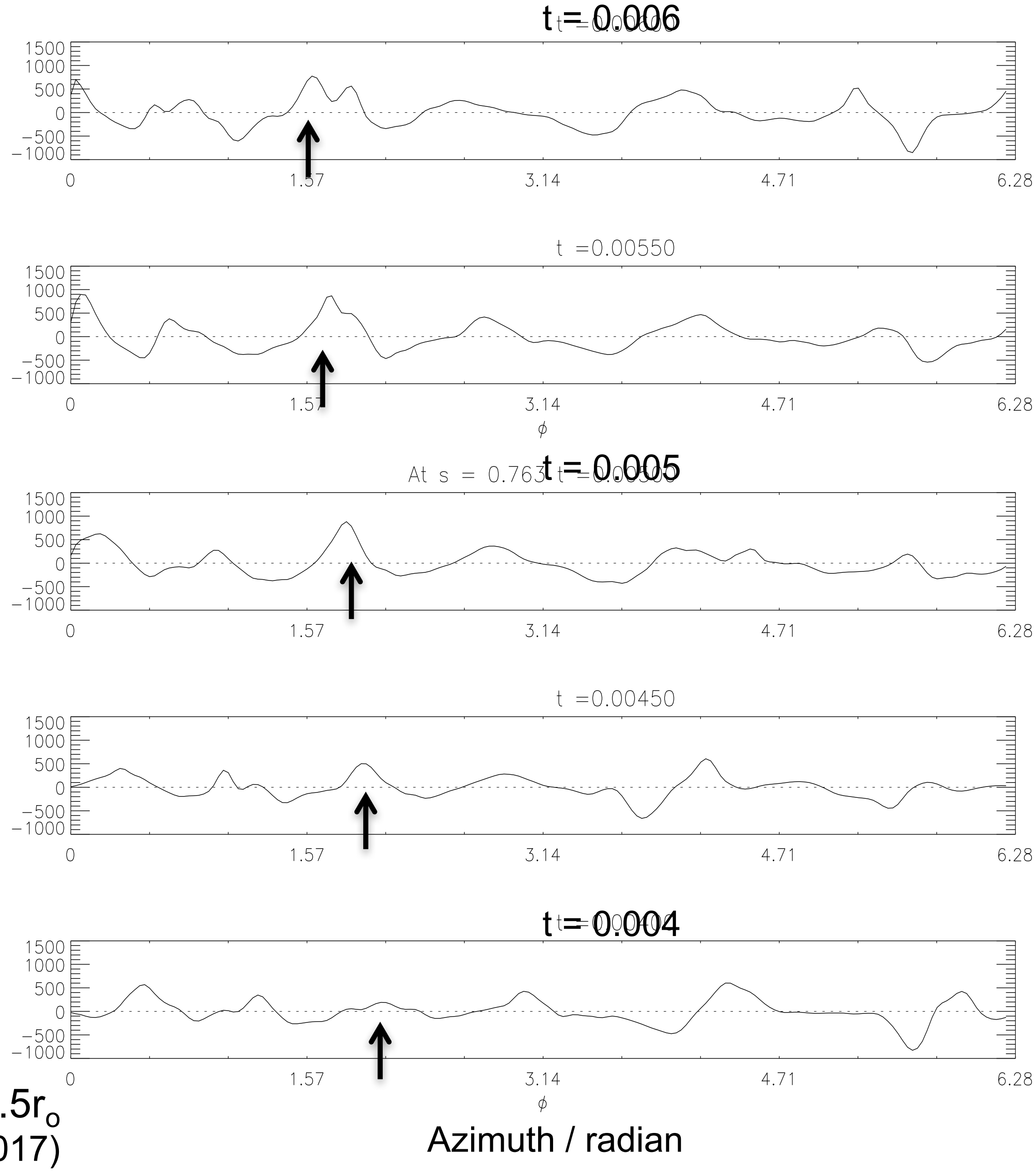


Nonlinearity on waveforms?

The observed waves illustrate

- no wave packets
- isolated, sharp waveforms
  - steepening
  - shifted to positive

- reminiscent of cnoidal/solitary waves in weakly nonlinear, dispersive waves (e.g. Whitham 1974)
  - cf. (nonmag) solitary Rossby (e.g. Redekkop 1977, Yamagata 1982)



Evolution of amplitude $<u_s'>$ at $s=0.5r_o$
(Hori, Teed & Jones 2017)

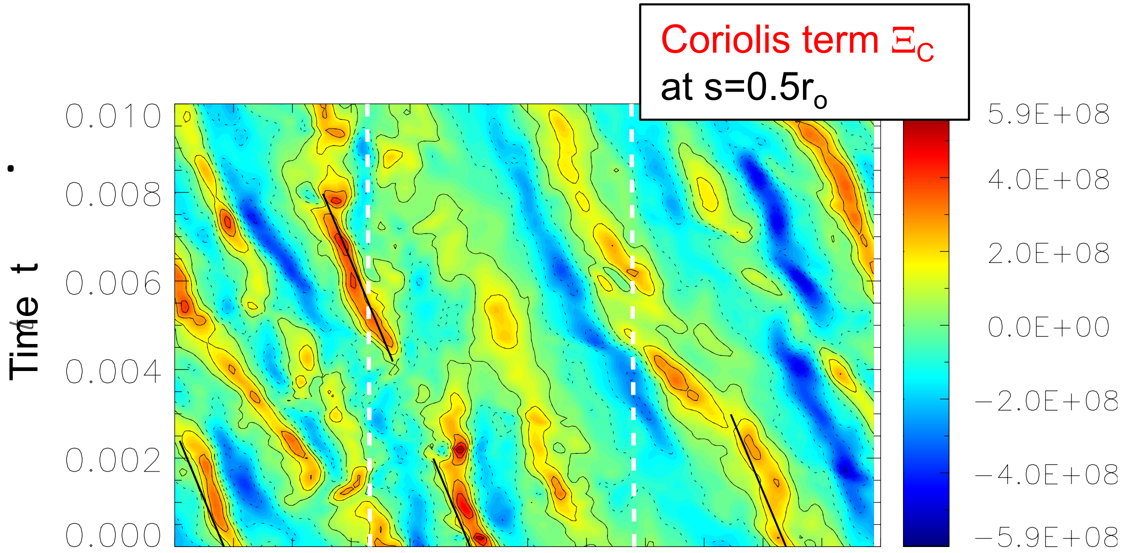
The role of nonlinear Lorentz force

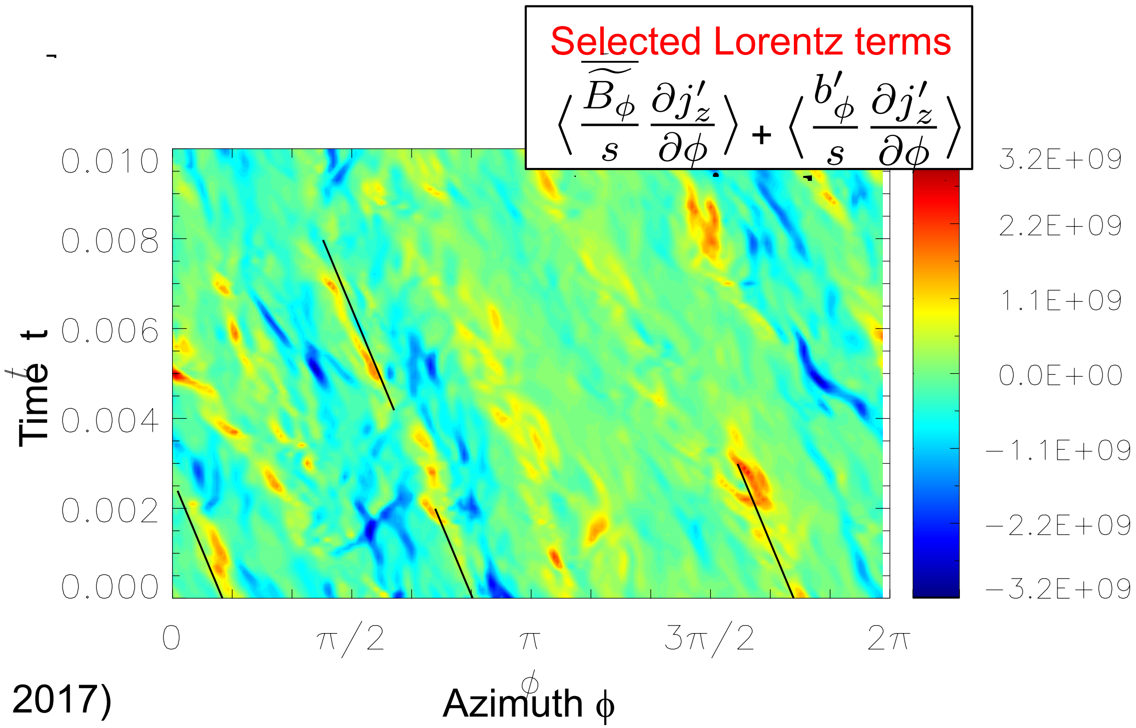
- Coriolis and Lorentz terms are dominant in the axial vorticity eq.
  - Reynolds term remains minor
- The Lorentz term Ξ_L can be expanded, in terms of the mean and fluctuating parts, as

$$\Xi_L = \frac{Pm}{E}\Big[\langle \widetilde{\overline{\boldsymbol{B}}} \cdot \nabla j'_z \rangle + \langle \boldsymbol{b}' \cdot \nabla j'_z \rangle$$

+ (other terms)]

  - first term for the restoring force
  - second term for the leading nonlinear part
- The sum of the dominant restoring and nonlinear terms reproduces steepened shapes





(Hori, Teed & Jones 2017)

Toroidal field strength within the Earth's core

- The dispersion relation tells us about waves riding on mean flow advection

$$\hat{\omega}_{M\beta} = \omega - \omega_{\mathrm{adv}} = -\frac{m^3(r_o^2 - s^2)\langle\overline{\widetilde{B_\phi^2}}\rangle}{2\rho\mu_0\Omega\, s^4}$$

  – a geomagnetic drift speed of 0.56 º/yr at 40º S (Finlay & Jackson 2003)

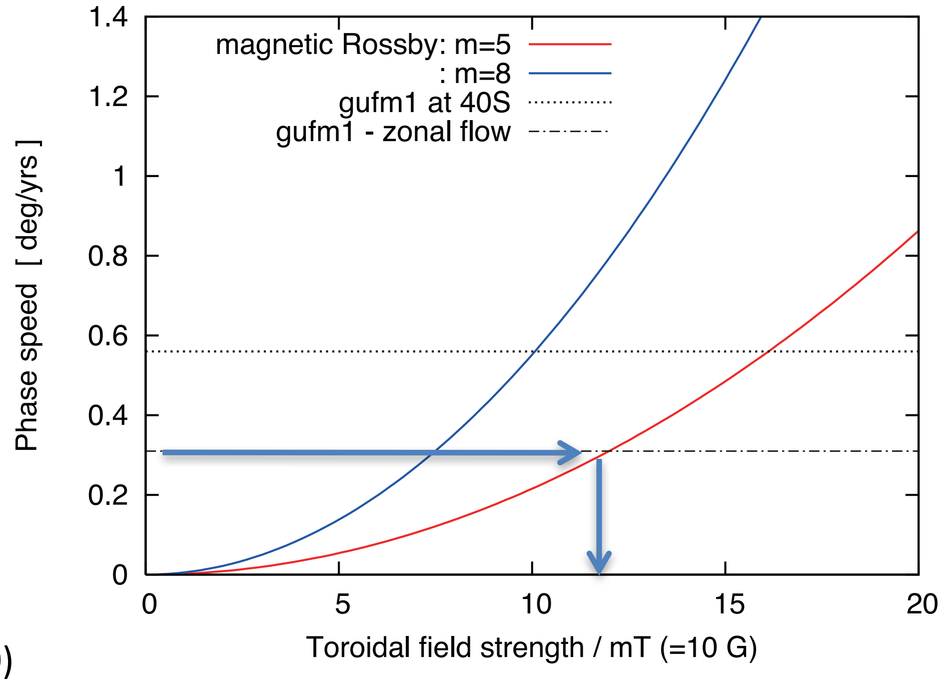
  – suppose a mean flow of 0.24 º/yr (Pais et al. 2015)

  – Given m=5, this implies a z-mean toroidal field B_ϕ ~ 12 mT at s ~ $0.8r_o$

    - equivalent to, or stronger than, the poloidal field Bs ≥ 3 mT (Gillet et al. 2010)

  – constrains the dynamo mechanism?

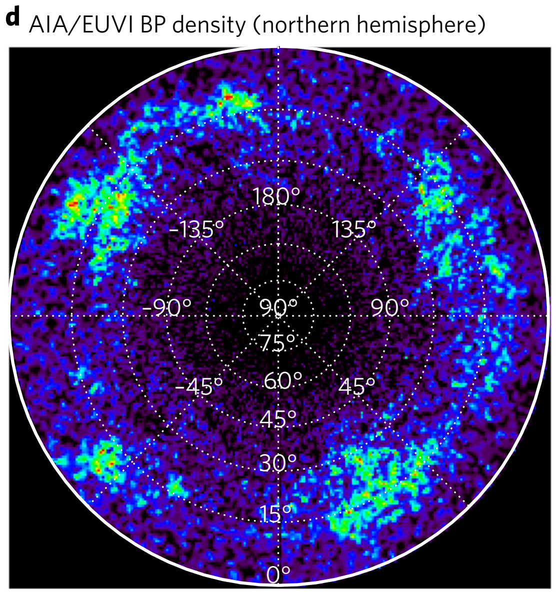
    - e.g. α^2-type or $\alpha\omega$ -type
    - stronger poloidal fields in dynamo simulations

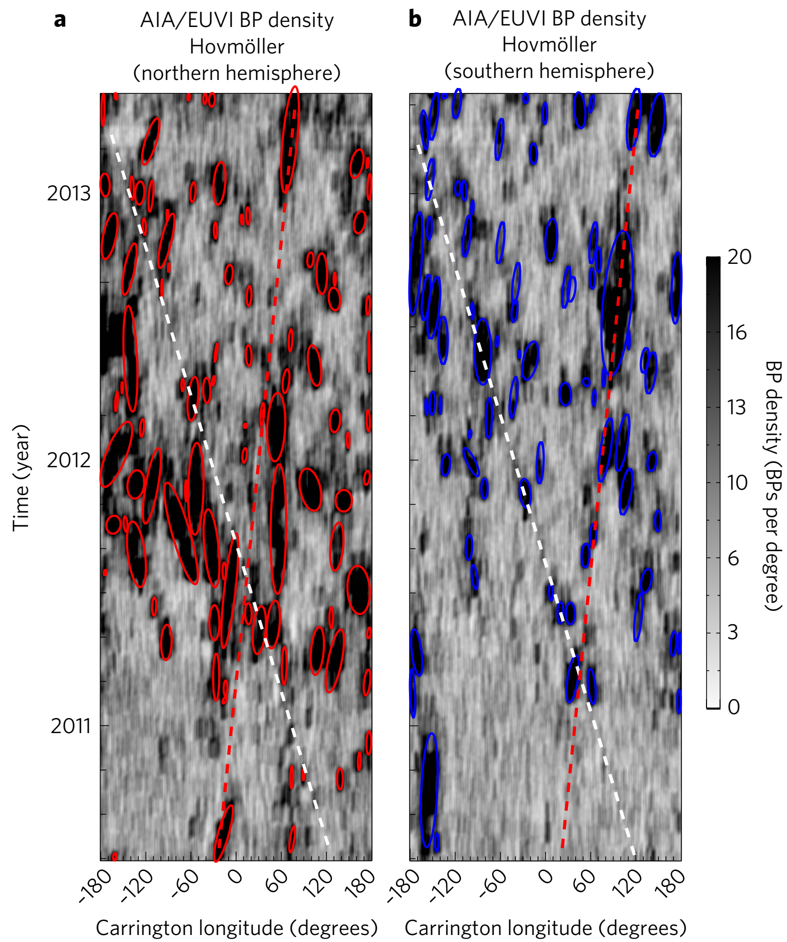


(Hori, Jones & Teed, 2015)

In thin, stably stratified layers

- A stable layer at the top of the Earth's core
  - SW models applied by poloidal field (Braginsky 1984, 1999)
- Solar tachocline at the bottom of the convection zone
  - SW models applied by toroidal field (Gilman 2000; Zaqarashvili et al. 2007)
  - ~ 3 m/s westward drifts and eastward wavetrains? (McIntosh et al. 2017)





Coronal brightpoints in Jan 2012 & at around 15° N / 22° S (McIntosh et al. 2017)

e.g. equatorial waves (cartesian)

- β-plane shallow water models applied by an azimuthal field

$$\frac{\partial u_x}{\partial t} - f u_y = \frac{B_x}{4\pi\rho}\frac{\partial b_x}{\partial x} - g\frac{\partial h}{\partial x}, \qquad \frac{\partial b_x}{\partial t} = B_x\frac{\partial u_x}{\partial x}, \quad \frac{\partial b_y}{\partial t} = B_x\frac{\partial u_y}{\partial x},$$

$$\frac{\partial u_y}{\partial t} + f u_x = \frac{B_x}{4\pi\rho}\frac{\partial b_y}{\partial x} - g\frac{\partial h}{\partial y}, \qquad \frac{\partial h}{\partial t} + H_0\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) = 0,$$

- when f ~ βy,

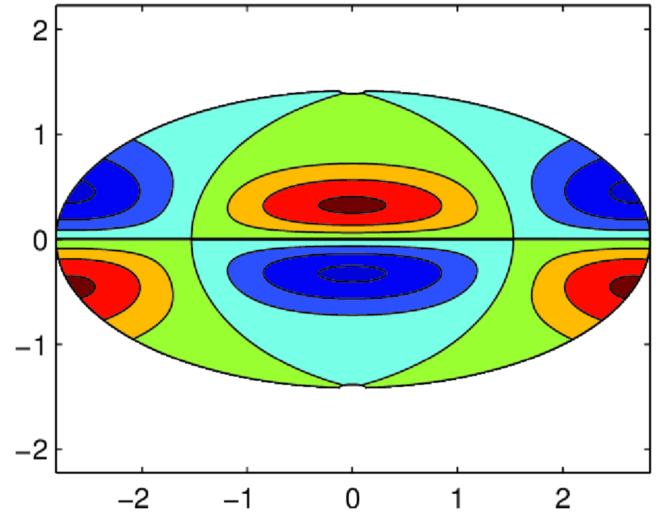
$$\frac{d^2 u_y}{dy^2} + \left[\frac{\omega^2}{C_0^2} - k_x^2\left(1 + \frac{v_A^2}{C_0^2}\right) - \frac{k_x \beta}{\omega(1 - k_x^2 v_A^2/\omega^2)} - \frac{\beta^2}{C_0^2(1 - k_x^2 v_A^2/\omega^2)} y^2\right] u_y = 0$$

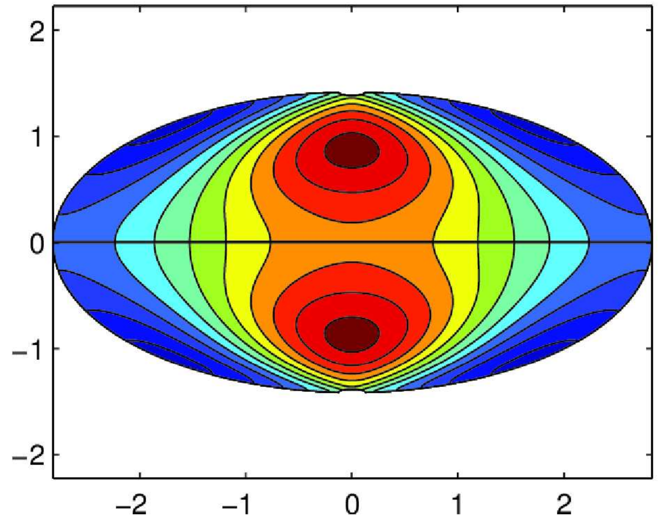
  – cf. nonmagnetic case (e.g. Matsuno 1966) :
    - a Schroedinger eq.
    - oscillatory for |y| < y_c, i.e. equatorially trapped waves
  – In the presence of magnetic field
    - nonzero V_A increases y_c, i.e. releasing the trapped waves
    - large V_A gives rise to a Bessel eq.

In spherical shells

- Nonaxisymmetric MAC waves classified:
  - inertio-gravity
  - Rossby
  - Kelvin

- Rossby: for eq.symmetric $B\phi = B0 \sin\theta$
  (Marquez-Artavia et al., 2017)
  - fast modes
    - goes westward
    - in the limit $V_M^2/V_c^2 << 1$, $\omega = -\frac{2\Omega_0 m}{n(n+1)}$
  - slow modes
    - goes eastward
    - in the limit $V_M^2/V_c^2 << 1$, $\omega = \frac{m v_a^2}{2\Omega_0 R_0^2}(n(n+1)-2)$
    - slowly westward for n=m=1
  - even polar trapped at large V_M^2/V_c^2
  - become unstable at large V_M^2/V_c^2





Eigenfunctions of fast / slow MR waves
for m=1, α ($\sim V_M^2/V_c^2$) = 0.1, ε^{-1} ($\sim V_A^2/V_c^2$) = 0.01

Summary

- Geo-/Jovian dynamo simulations are supporting the excitation of magnetic Rossby waves for incompressible/ anelastic fluids
  - crests/troughs going retrogradely on timescales of O($10^{1\text{-}2}$ yrs) in the Earth’s core, about mean zonal flows
  - excited when torsional Alfven waves were excited
    - for strong-field dynamos (Pm >=5 or E =< 10^{-4}; Λ >~ 2)
  - the speeds accounted for by the linear theory, but their waveforms steepened, likely due to nonlinear Lorentz terms
  - their speeds potentially revealing the strength of the ‘hidden’ toroidal field
  - induced by topography but also by compressibility

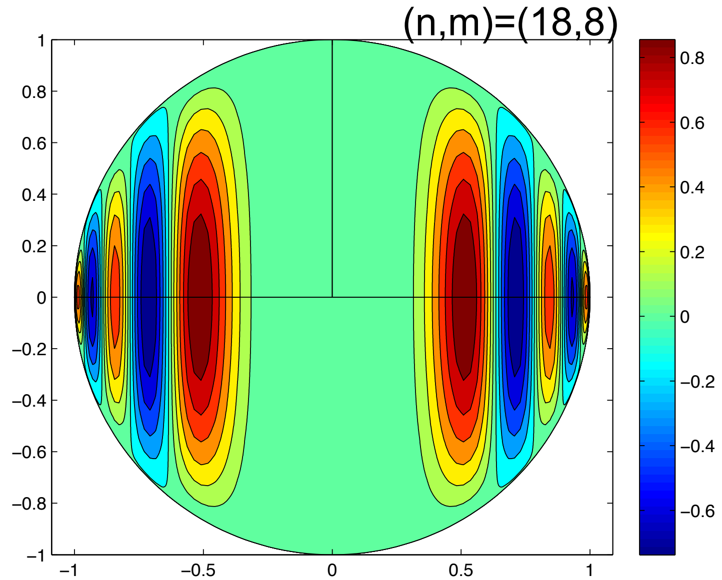
Thank you

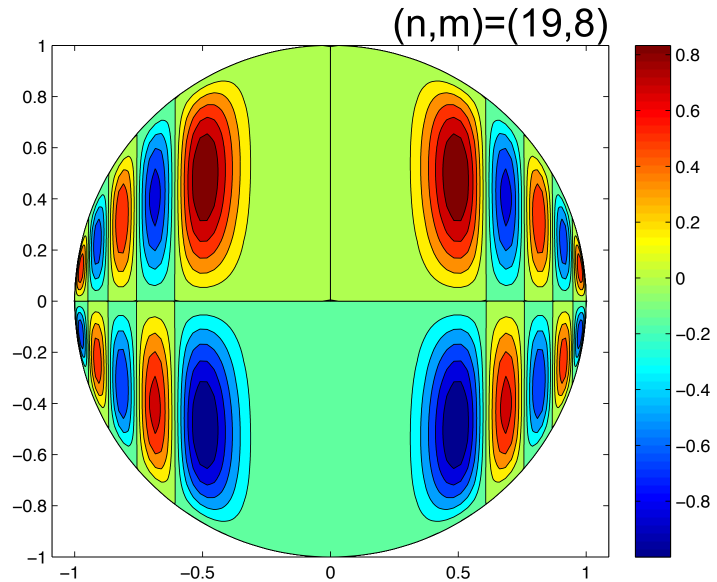
QG vs. non-QG modes

- In spheres
  - e.g. for Malkus field (1967)

    $B_\phi = B_0\, s\, e_\phi$

  - the solution, $P=P_n^m(\mu)\, P_n^m(\mu)$
  - equatorially trapped for for small n
  - even (n-m): eq.symmetric (QG) modes
    - goes retrograde & faster ($\approx -\omega_M^2/\omega_\beta$)
  - odd (n-m): eq.anti-symmetric modes
    - goes prograde & slower ($\approx +\omega_M^2/\omega_I$)
  - cf. MC waves in simple plane layers
    - slow modes has no preference in propagation direction ($\approx \pm\omega_M^2/\omega_C$)
    - The geometrical effect splits the modes into a faster & slower ones





Eigenfunctions for Malkus field (after Malkus 1967)