

## Hamiltonian Fluid Mechanics

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This will be an *elementary introduction* following my book:

*Lectures on Geophysical Fluid Dynamics* [LGFD], Oxford, 1998  
Chapter 1 (pp 1-12), Chapter 4 (pp 197-206), **Chapter 7**

### Schedule of Lectures

#### Tuesday

9:00 Hamilton's principle & the particle-relabeling symmetry property  
14:00 Approximations that retain conservation laws

#### Wednesday

9:00 Mean flows & disturbances

#### Thursday

9:00 Eulerian variational principles & Poisson brackets  
14:00 Pseudoenergy, stability, and available energy

### References

#### General

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References for the seminar on numerical methods

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References for lecture 1: Hamilton's principle and the particle-relabeling symmetry property

Salmon, LGFD, pp 1-12, 197-206, 295-304.

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References for lecture 2: Approximations that retain conservation laws

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References for lecture 3: Mean flows and disturbances

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References for lecture 4: Eulerian variational principles & Poisson brackets

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References for lecture 5: Pseudoenergy, stability, and available energy

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