CHIBA UNIVERSITY

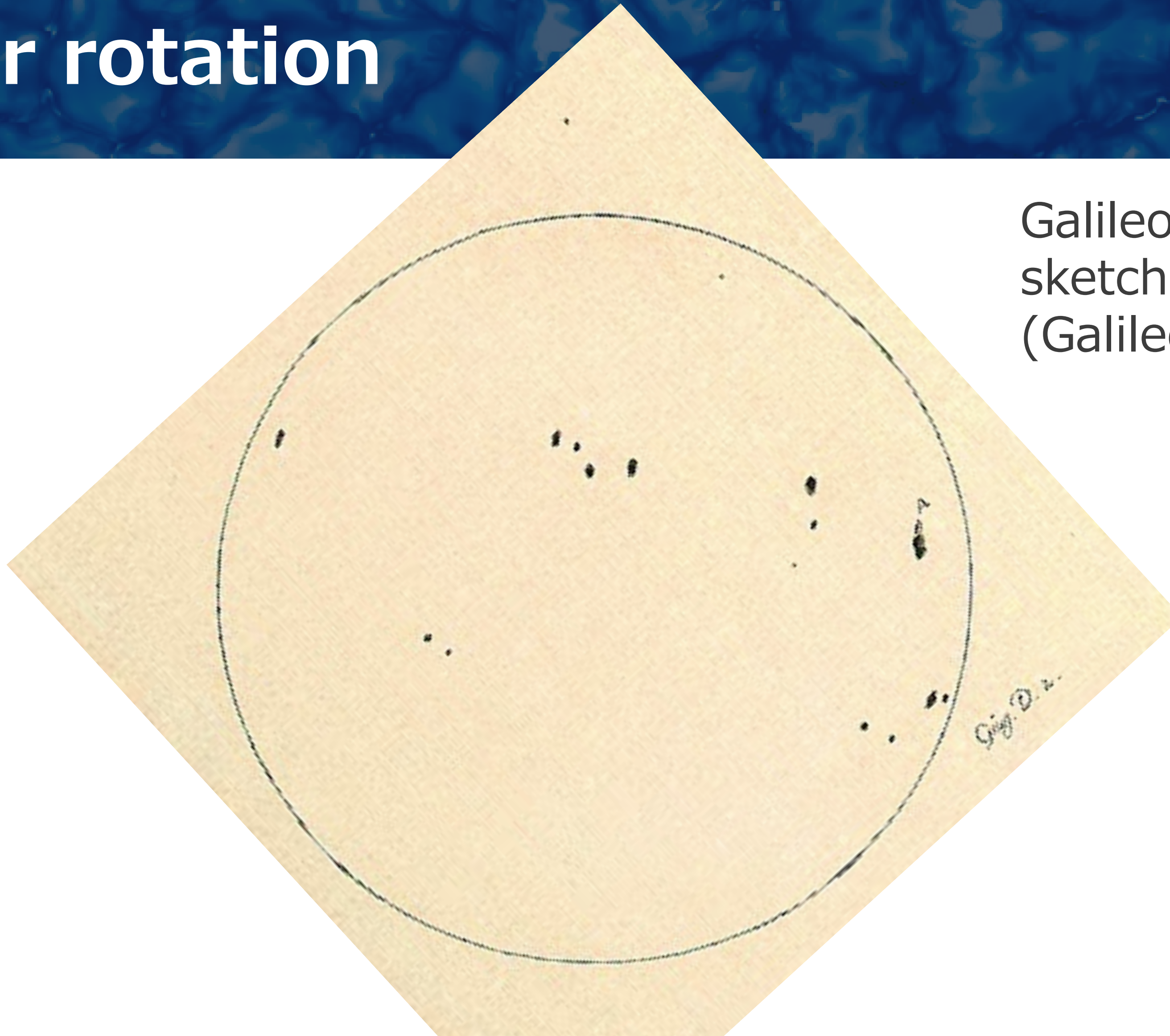
2022年2月9日
GFD/CPS セミナー

高解像シミュレーションで迫る太陽赤道加速の謎

Hotta & Kusano, 2021, Nature Astronomy, 5, 1100
Hotta, Kusano & Shimada, 2022, ApJ, 933, 199,

堀田英之(千葉大学)
草野完也(名古屋大学)、嶌田遼太(東京大学)

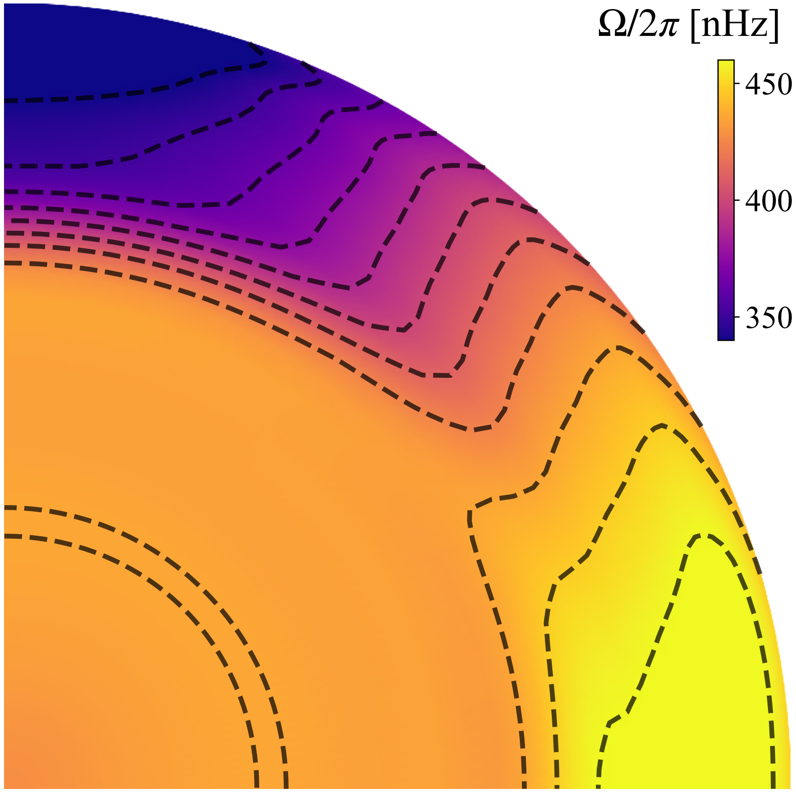
Solar rotation



Galileo's sunspot sketch ~1620s (Galileo project)

The sun is rotating. This is first understood with the sunspot tracking.
Around 1630, we also begin to know that the sun is differentially rotating.

Solar differential rotation



Data provided by R. Howe
Result of helioseismology

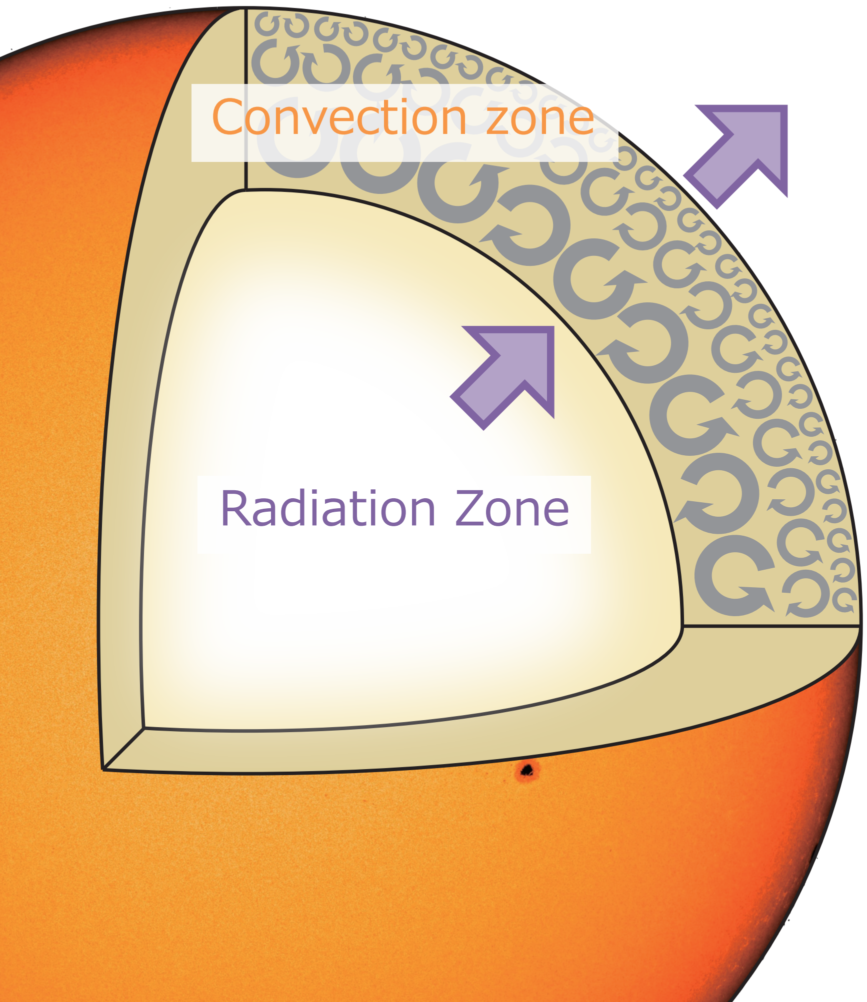
The solar differential rotation is precisely evaluated by the helioseismology (error < 1%).

Differential rotation is a fundamental ingredient of solar dynamo.

We know several interesting features.

A. Fast equator, and slow pole
B. Tachocline
C. Conical profile in the middle CZ
D. Near surface shear layer

Solar interior and convection



Energy generated by the nuclear fusion in the solar core is transported by

✓ Radiation (inner 70%)
✓ Convection (outer 30%)

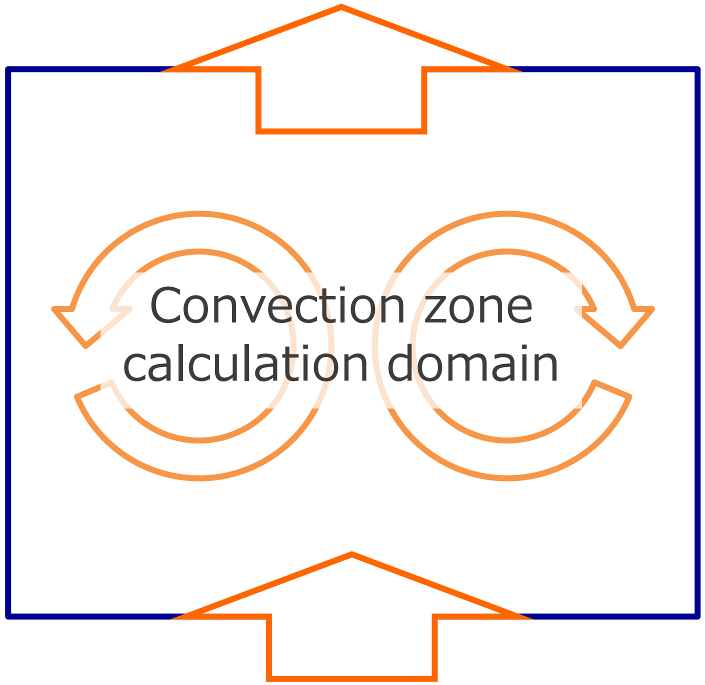
The turbulent motion of the ionized plasma can interact with magnetic field. Anisotropic turbulence and magnetic field causes the angular momentum transport and results in the differential rotation.

The turbulence in the Sun is highly chaotic and numerical simulation is an essential approach.

Solar convection calculation

$$L_\odot = 3.84 \times 10^{33} \text{ erg s}^{-1}$$

Radiation energy flux



Radiation energy flux
(well known value)

We have precisely evaluated the radiation energy flux from the solar surface.

The energy flux is imposed at the bottom boundary and extracted from the top boundary.

The stratification is also well known by the solar model confirmed with the helioseismology.

There is almost no ambiguity and the "correct" calculation should lead to the correct flow and the magnetic field.

Equations for the solar convection zone

Magnetohydrodynamics (MHD)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v})$$

Mass conservation

Gravity

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) = -\nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) - \nabla p \boxed{+ \rho \boldsymbol{g}}$$

Momentum conservation

Lorentz force

$$\boxed{+\frac{1}{4\pi}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}} \boxed{+ 2\rho \boldsymbol{v} \times \boldsymbol{\Omega}}$$

Coriolis force

$$\rho T \frac{\partial s}{\partial t} = -\rho T (\boldsymbol{v} \cdot \nabla) s + \boxed{Q_{\mathrm{rad}}}$$

Entropy equation
or energy conservation

Radiation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

Magnetic induction equation

$$p = p(\rho, s)$$

Equation of state (including ionization)

Angular momentum transport

Turbulence can transport the angular momentum. $\mathcal{L} = \lambda u_\phi$ is the specific angular momentum and $\lambda = r\sin\theta$.
Velocities are divided to mean $\langle v \rangle$ and perturbed v' parts. $v = \langle v \rangle + v'$
Then the angular momentum conservation equation is ($\boldsymbol{B}$ is ignored)

$$
\begin{aligned}
\frac{\partial}{\partial t}\left(\rho_0 \langle \mathcal{L} \rangle\right) &= -\nabla \cdot \langle \rho_0 \boldsymbol{v}_\mathrm{m} \mathcal{L} \rangle \\
&= -\nabla \cdot \left(\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \langle \mathcal{L} \rangle\right) - \nabla \cdot \left(\rho_0 \lambda \langle \boldsymbol{v}'_\mathrm{m} v'_\phi \rangle\right) \\
&= \underbrace{-\left(\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \cdot \nabla\right) \langle \mathcal{L} \rangle}_{\mathrm{transport\ by\ meridional\ flow}} \quad \underbrace{-\nabla \cdot \left(\rho_0 \lambda \langle \boldsymbol{v}'_\mathrm{m} v'_\phi \rangle\right)}_{\mathrm{transport\ by\ turbulence}}
\end{aligned}
$$

There are two contributions of the angular momentum transport transport by the meridional flow and turbulence (Reynolds stress)

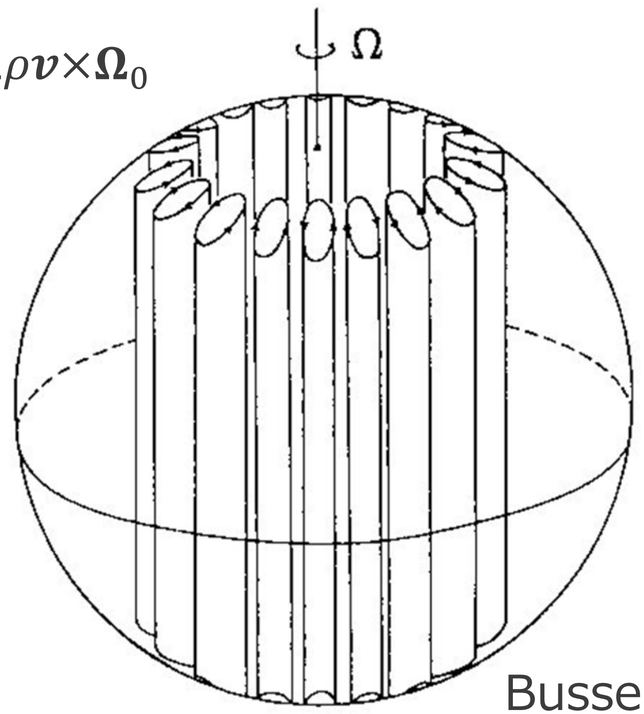
The velocity correlation $\langle v'_i v'_j \rangle$ is important to understand the solar differential rotation.

How to determine the differential rotation

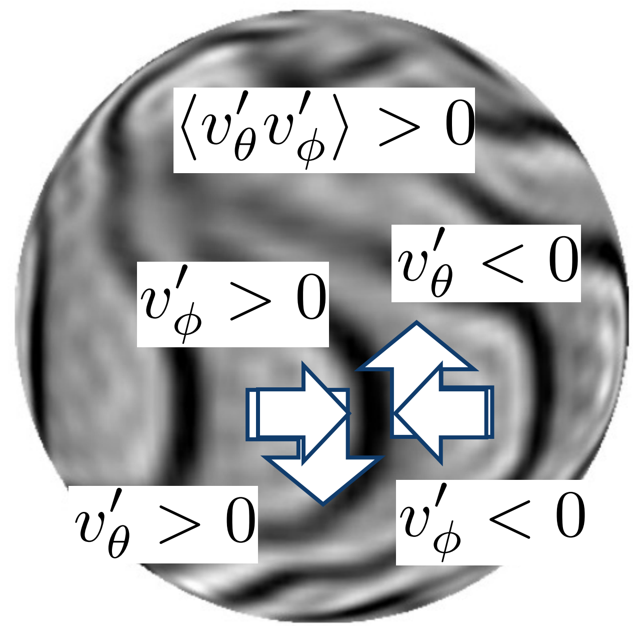
Angular momentum conservation

$$\rho_0 \frac{\partial \langle \mathcal{L} \rangle}{\partial t} = -\rho_0 \langle \boldsymbol{v}_{\mathrm{m}} \rangle \cdot \nabla \langle \mathcal{L} \rangle - \nabla \cdot \left(\rho_0 \lambda \langle \boldsymbol{v}'_m v'_\phi \rangle \right)$$

$2\rho\boldsymbol{v}\times\boldsymbol{\Omega}_0$



Busse, 1970

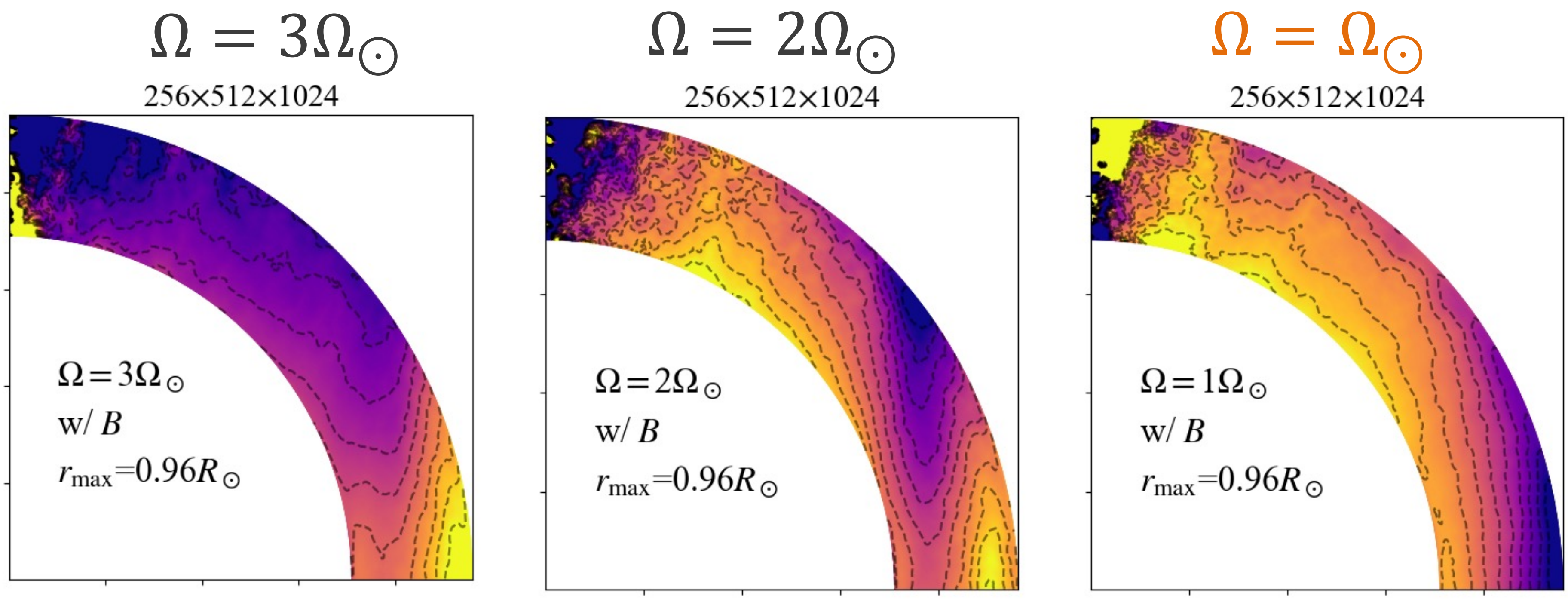


Miesch+2000

Rotational influence cause equatorward angular momentum transport and suppresses poleward one.

Convective conundrum

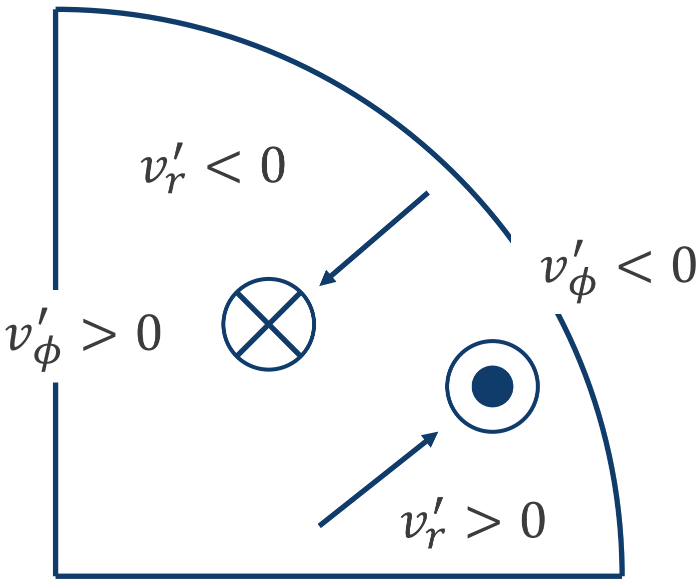
Typical "high resolution" simulation for the sun fails to reproduce the solar like differential rotation

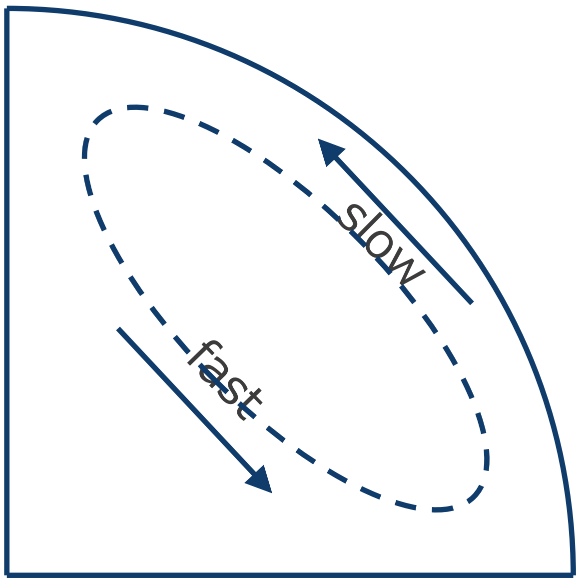


We probably fail to reproduce the convection and/or the magnetic field in the convection zone to construct the solar-like differential rotation.

Slow rotation case (1/2)

In a slow rotation case, the convection itself is prominent. The origin of the angular momentum transport is the radial velocity v_r





$$\langle v'_r v'_\phi \rangle < 0$$

Radially inward angular momentum transport

$$\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \cdot \nabla \langle \mathcal{L} \rangle = -\nabla \cdot \left(\rho_0 \lambda \langle \boldsymbol{v}'_m v'_\phi \rangle \right)$$

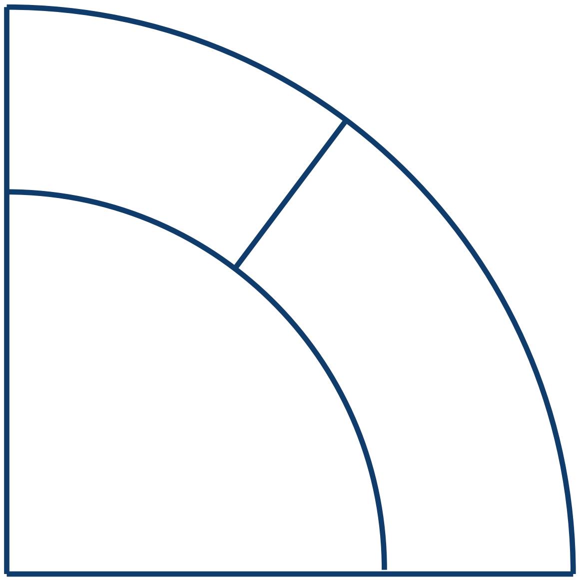
The angular momentum transport causes anti-clockwise meridional flow.

Slow rotation case (2/2)

Important slide

Anti-clockwise meridional flow always transport the AM poleward.
The colatitudinal AM by the meridional flow is:
$\rho_0\langle v_\theta\rangle\langle\mathcal{L}\rangle$, where $\mathcal{L} = r\sin\theta\, u_\phi$.

In addition, due to the extremely low Mach number the fluid satisfies the anelastic approximation $\nabla\cdot(\rho_0\boldsymbol{v}_\mathrm{m}) = 0$, which lead to $\int\rho_0\langle v_\theta\rangle r dr = 0$ at constant θ surface.



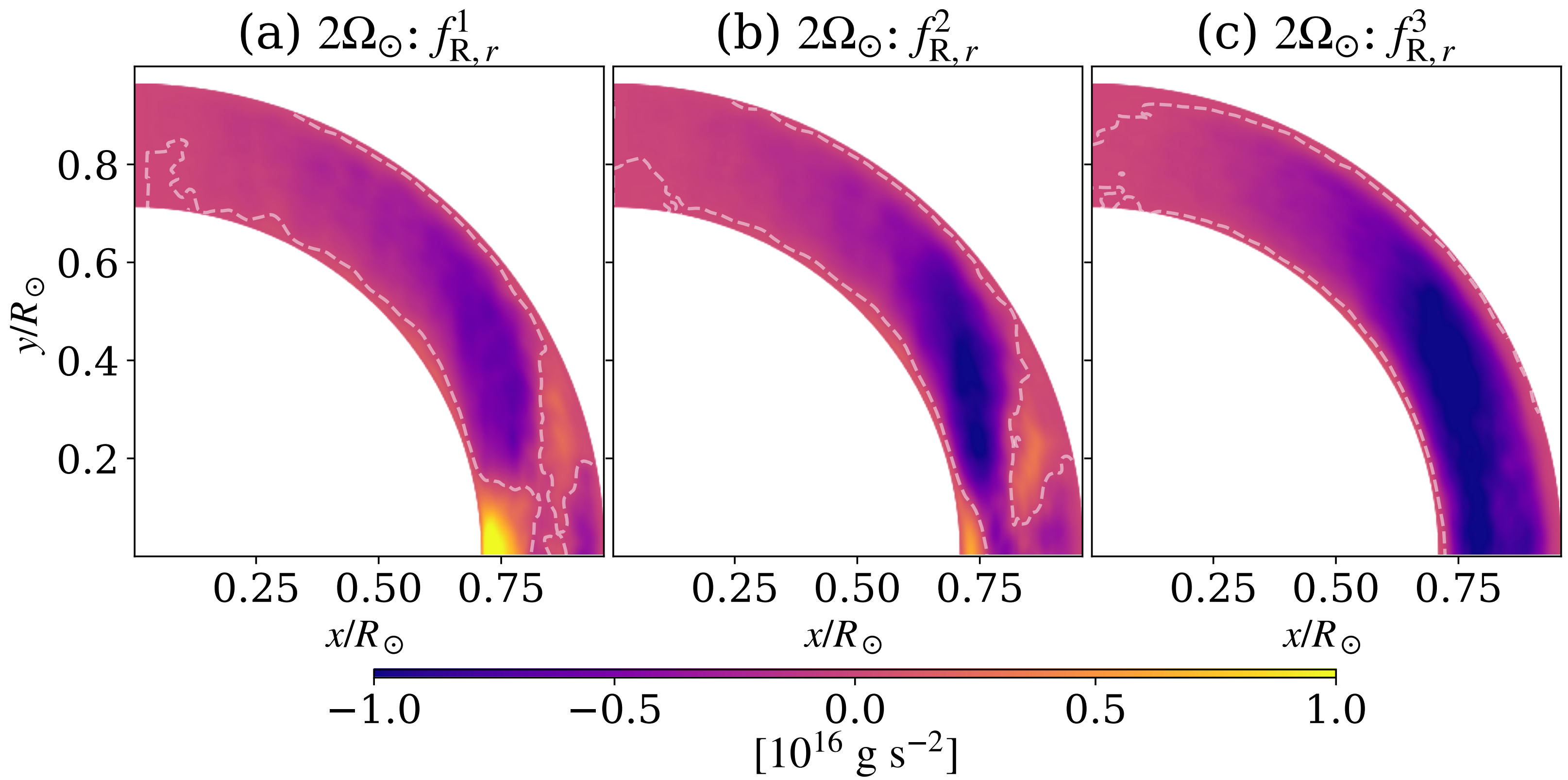
Thus, the net AM transport $\int\rho_0\langle v_\theta\rangle\langle\mathcal{L}\rangle r dr$ is determined by the specific AM distribution, here Since the solar differential rotation is not strong $\Omega\sim\Omega_0$ and the deeper layer (small r) has smaller AM $\mathcal{L}\sim r^2\sin^2\theta\,\Omega_0$.

Thus, the anti-clockwise meridional flow MUST transports AM poleward.
We need to suppress negative $\langle v_r' v_\phi'\rangle$ to have the fast equator.

Scale dependence of AM flux

Radial angular momentum flux $\rho_0 r \sin\theta \left\langle v_r' v_\phi' \right\rangle$

$240\ \mathrm{Mm} < L$ $120\ \mathrm{Mm} < L < 240\ \mathrm{mm}$ $60\ \mathrm{Mm} < L < 120\ \mathrm{mm}$



Smaller scales tend to transport the AM radially inward

Mori & Hotta, 2023

Methods to reproduce the solar-like DR

$$\mathrm{Ro} = \frac{v}{2\Omega_0 L}$$

Large Ro → Fast pole
Small Ro → Fast equator

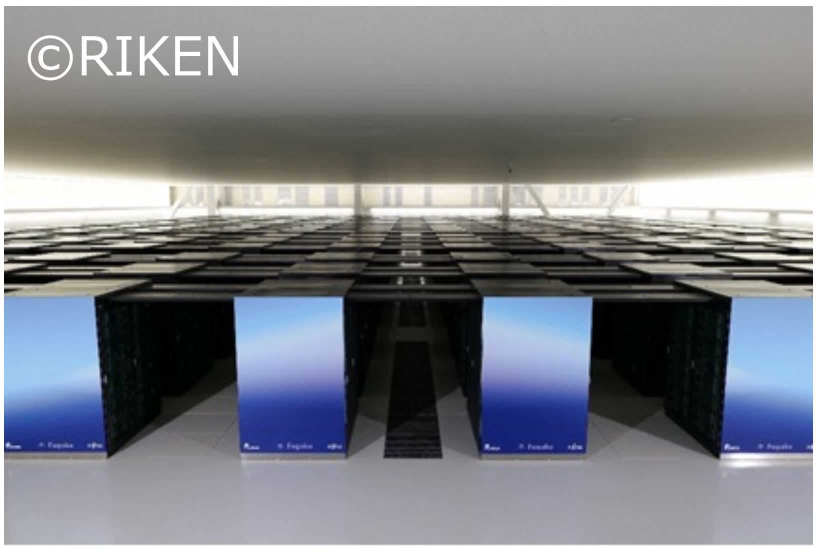
1. Increase angular velocity Ω_0
   Brown+2008, Nelson+2013, Hotta, 2018

2. Decrease luminosity $L_\odot$ → Decrease convection velocity
   Hotta+2015

3. Increase viscosity and/or thermal conductivity
   Miesch+2000, 2006, 2008, Brun+2002, 2004, Fan+2014, Hotta+2016

Angular velocity Ω_0 and luminosity $L_\odot$ are precisely observed values and we should not change these for solar simulations.
Large viscosity/thermal conductivity are not realistic
→ One of most important problems in solar physics

Fugaku



Top 500 list (Nov, 2021)

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	**Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu RIKEN Center for Computational Science Japan	7,630,848	442,010.0	537,212.0	29,899
2	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM DOE/SC/Oak Ridge National Laboratory United States	2,414,592	148,600.0	200,794.9	10,096
3	**Sierra** - IBM Power System AC922, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM / NVIDIA / Mellanox DOE/NNSA/LLNL United States	1,572,480	94,640.0	125,712.0	7,438
4	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway, NRCPC National Supercomputing Center in Wuxi China	10,649,600	93,014.6	125,435.9	15,371
5	**Perlmutter** - HPE Cray EX235n, AMD EPYC 7763 64C 2.45GHz, NVIDIA A100 SXM4 40 GB, Slingshot-10, HPE DOE/SC/LBNL/NERSC United States	761,856	70,870.0	93,750.0	2,589

Theoretical peak exceeds 500 PFLOPS with boost mode.
48 cores/node × 158976 node = 7630848 cores
CPU: A64FX Armv8.2-A SVE (512 bit SIMD)
We use Fugaku to attack the convective conundrum!

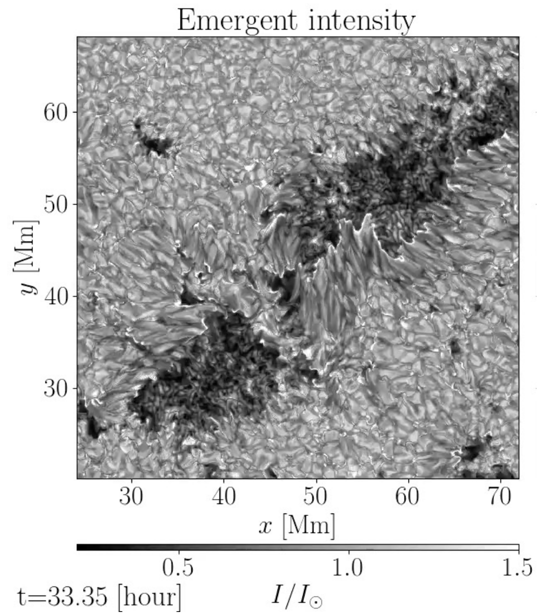
R2D2 code

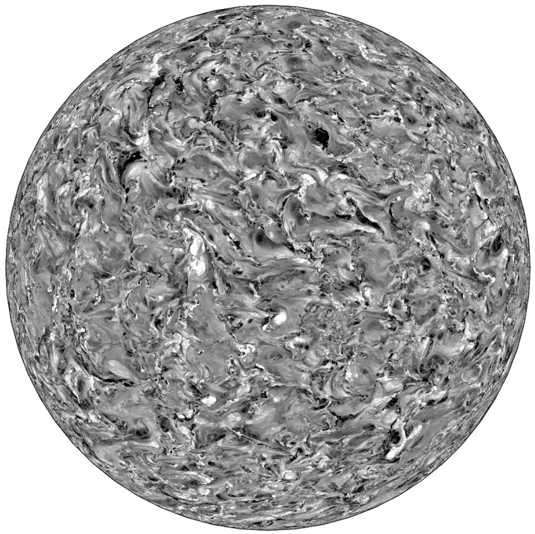
Radiation and RSST for Deep Dynamics

Hotta+2019, 2020a,b

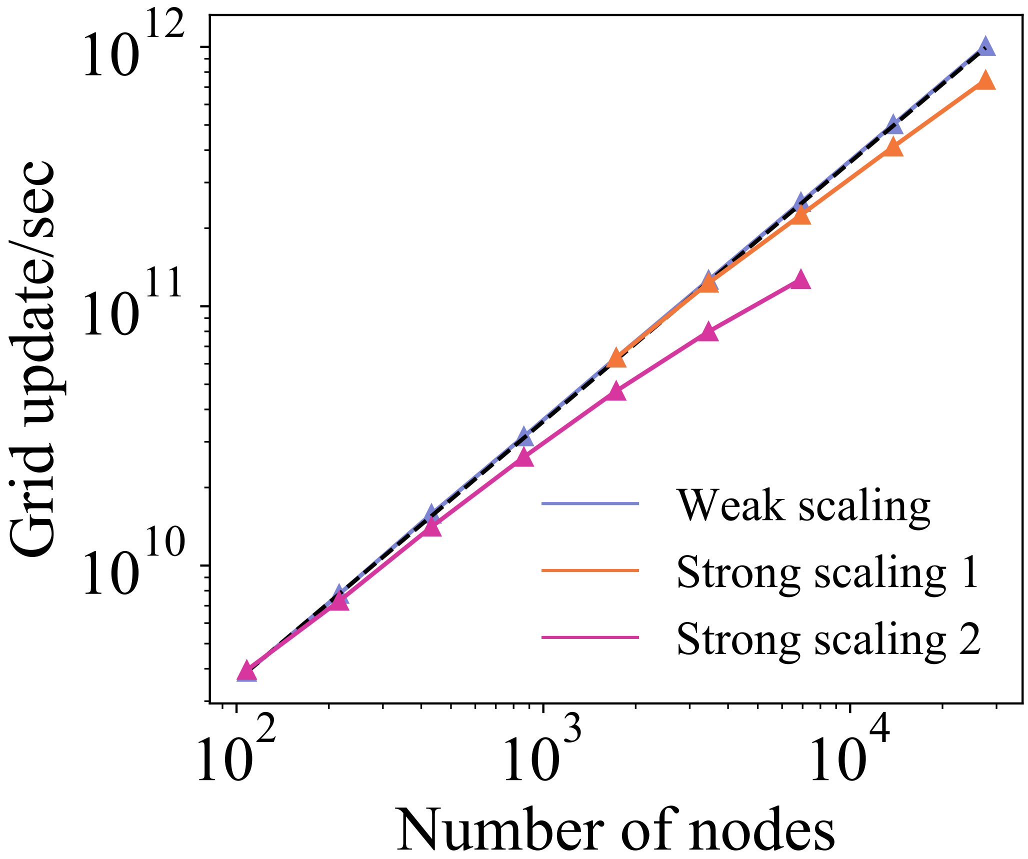
- ✓ 4th order accurate derivative (ununiform grid applicable)
- ✓ 4-step Runge-Kutta
- ✓ Cartesian・Spherical (Yin-Yang, Kageyama+2002)
- ✓ Non-linear artificial viscosity (Rempel, 2014)
- ✓ Entropy equation for deep convection zone
- ✓ Realistic radiation transfer
- ✓ OPAL EoS, Linear ⇄ Table
- ✓ Reduced Speed of Sound Technique (Hotta+2012, 2015)
- ✓ Alfven speed suppression (Rempel+2009)

In this study, abilities only for deep CZ is used.



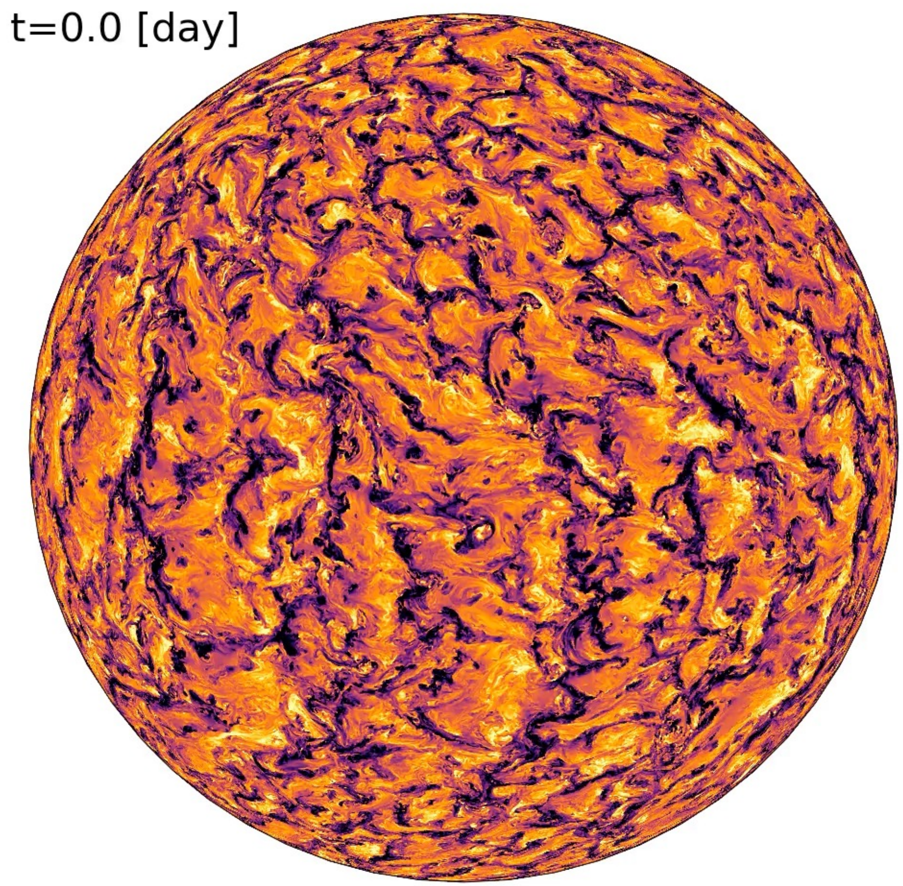


Performance of R2D2 on Fugaku



Almost perfect (>99%) weak scaling is checked more than 1M cores.
48 cores/node. We can carry out "large" calculation.
3.7×10^7 grid update/sec/node is achieved (~3 TFLOPS/node).

Numerical setting



Vertical velocity at $r = 0.9R_\odot$

Calculation domain:

$$0.71R_\odot < r < 0.96R_\odot$$

Number of grid points:

Low	96×768×1536
Middle	192×1536×3072
High	384×3072×6144

Calculation：4000 days
3.5 Mstep for High case.
Statistically steady flow is obtained around t=3000 day.

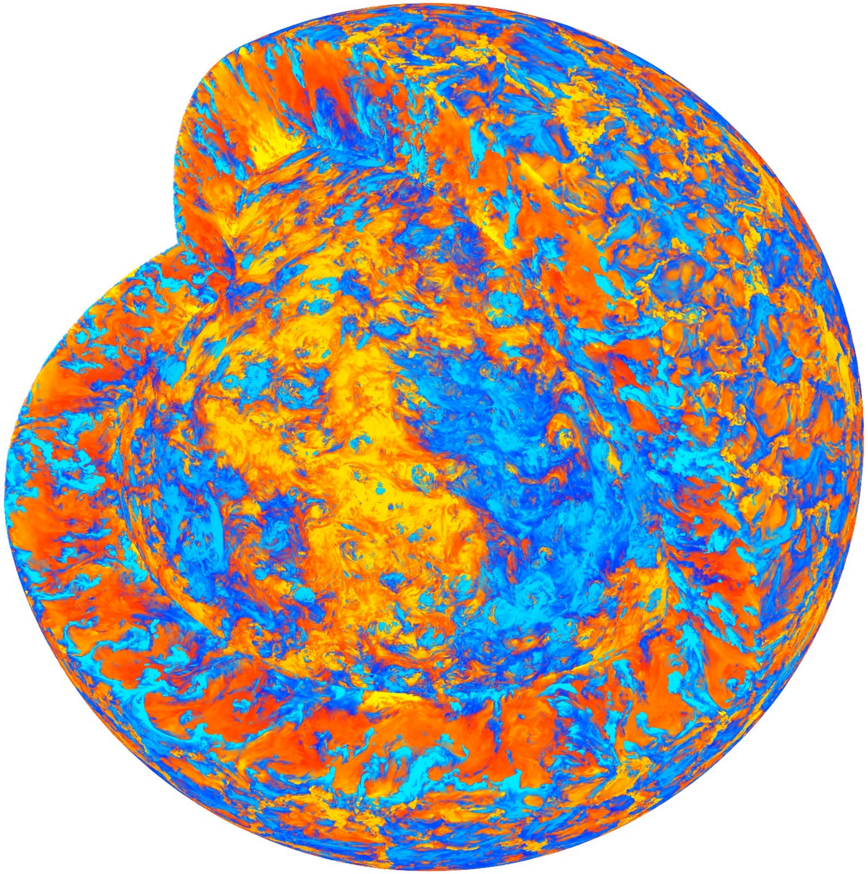
Data is averaged between 3600-4000 day to show the results.

No explicit diffusivities with $1\Omega_\odot$ and $1L_\odot$.

Convection and magnetic field

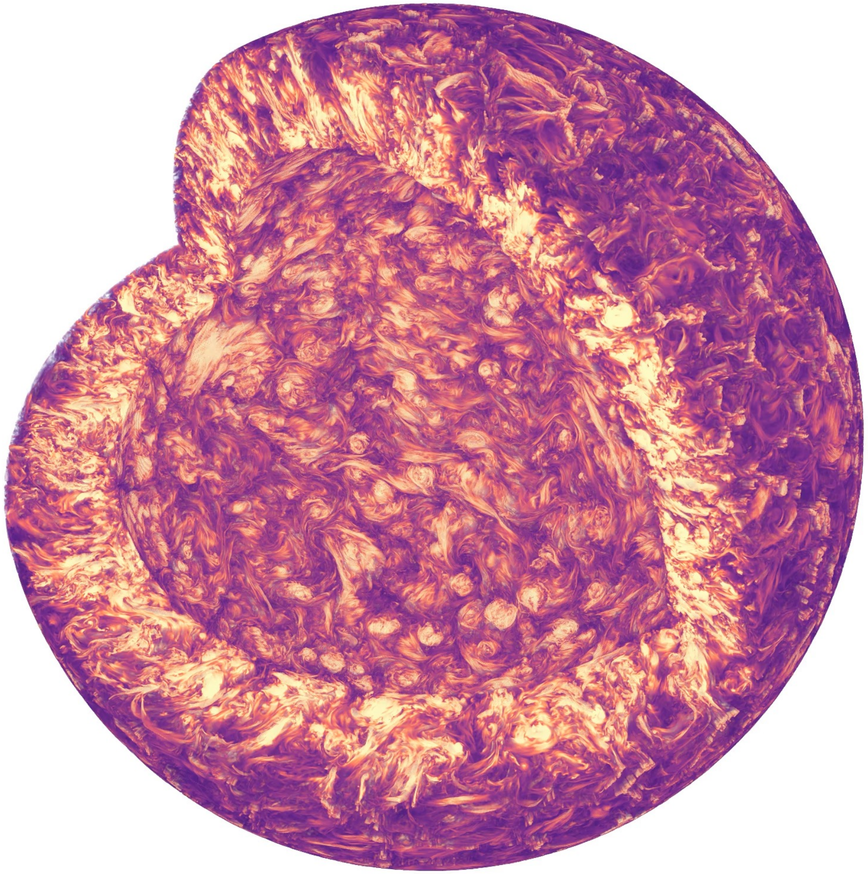
Normalized entropy

$(s - \langle s \rangle)/s_{\rm rms}$



Magnetic field strength

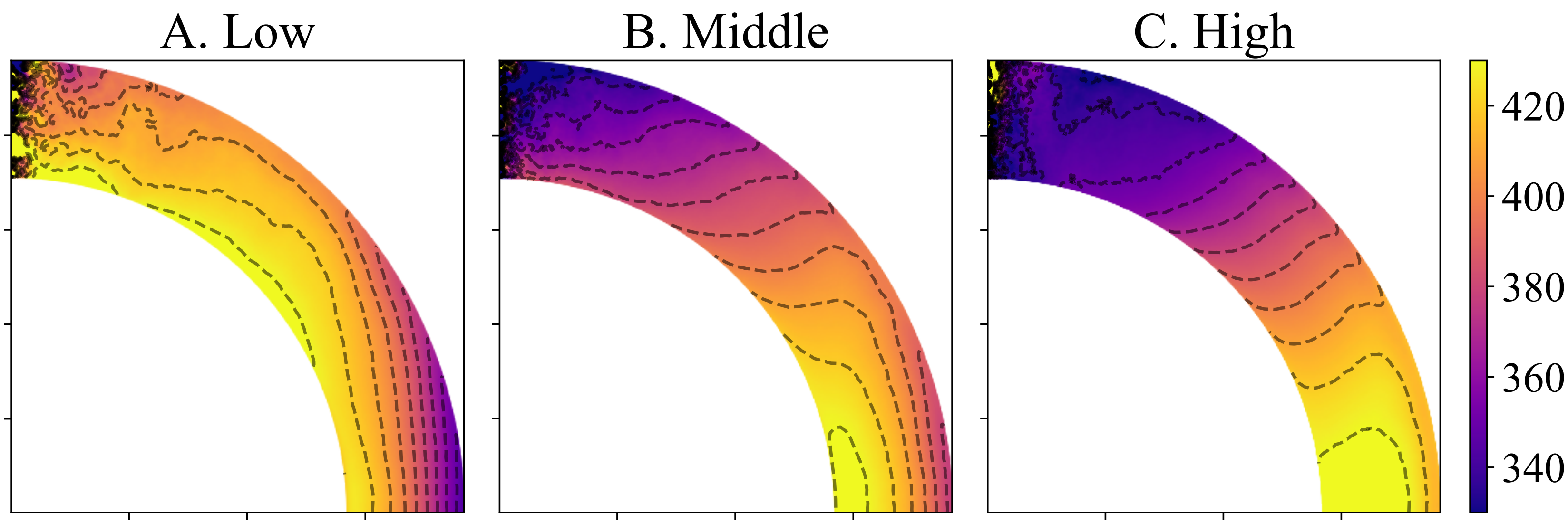
$|B|$ [kG]



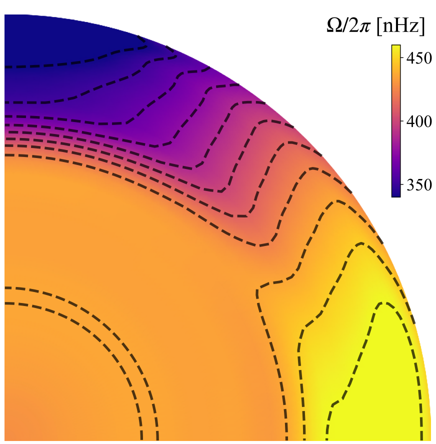
Dependence of DR on resolution

Increase resolution

96×768×1536　　192×1536×3072　　384×3072×6144

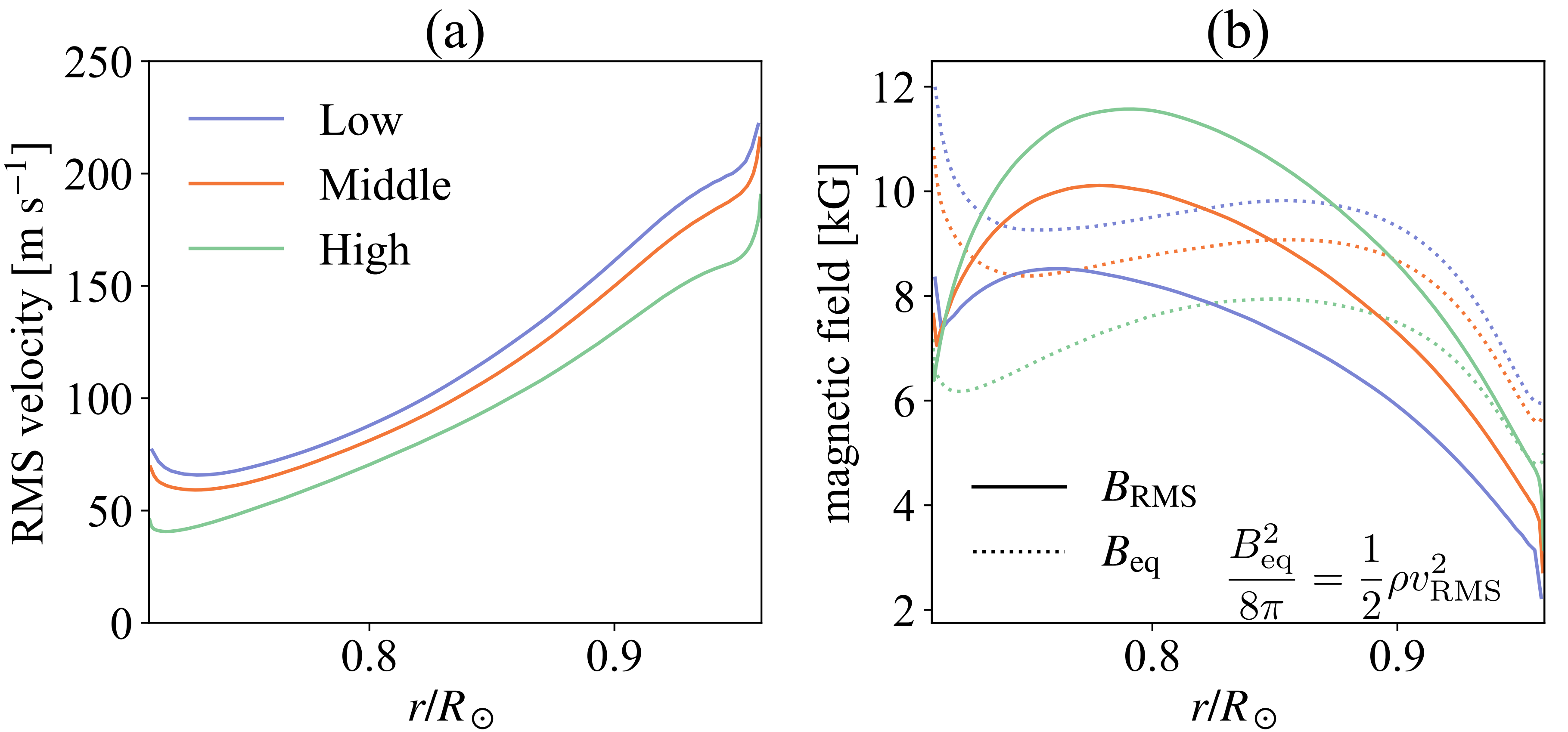


Sun



While the fast equator is obtained in the Low case (consistent with previous studies), the High case reproduces the solar-like differential rotation.

Convection and magnetic field



Increasing the resolution decreases the convection velocity and amplifies the magnetic field. The Low case show $E_{\rm kin} > E_{\rm mag}$ in all the depth, but the High case achieved $E_{\rm kin} < E_{\rm mag}$ throughout the convection zone. Situation is totally different.

Analyses of calculation

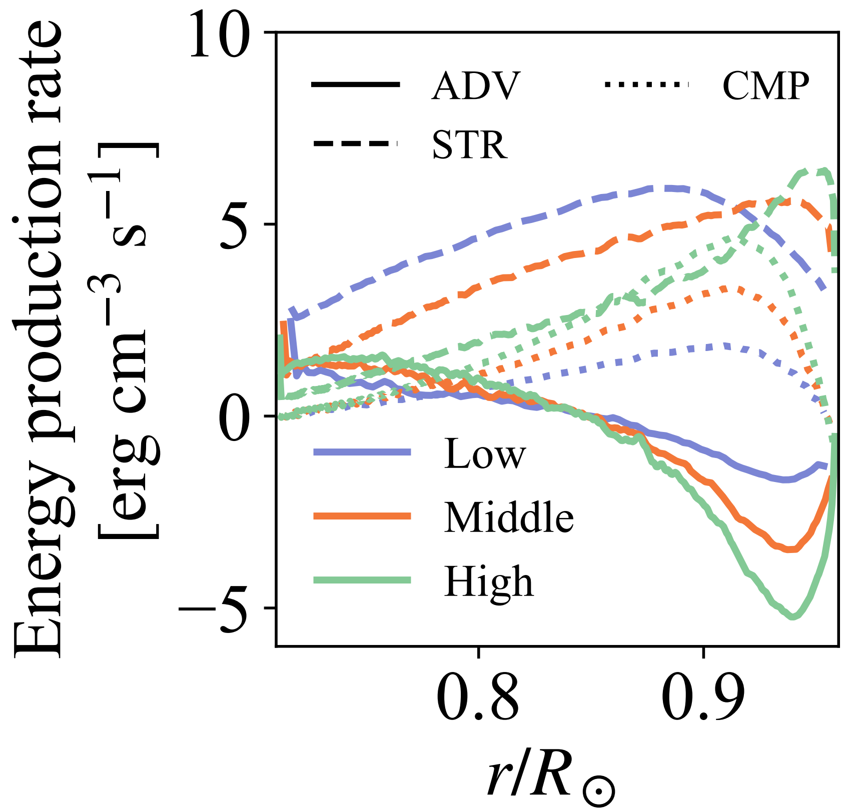
We analyze the huge data (>100 TB in High case), to understand the mechanism to maintain the solar-like differential rotation.

Key questions are:

1. Why the superequipartition magnetic field ($E_{\mathrm{mag}} > E_{\mathrm{kin}}$)?
2. Why the equator is rotating faster?

Magnetic field generation (1/2)

$$\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right) = \underbrace{-\frac{\boldsymbol{B}}{4\pi}\cdot\left[\left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{B}\right]}_{\mathrm{ADV}} \underbrace{+\frac{\boldsymbol{B}}{4\pi}\cdot\left[\left(\boldsymbol{B}\cdot\nabla\right)\boldsymbol{v}\right]}_{\mathrm{STR}} \underbrace{-\frac{B^2}{8\pi}\left(\nabla\cdot\boldsymbol{v}\right)}_{\mathrm{CMP}}$$

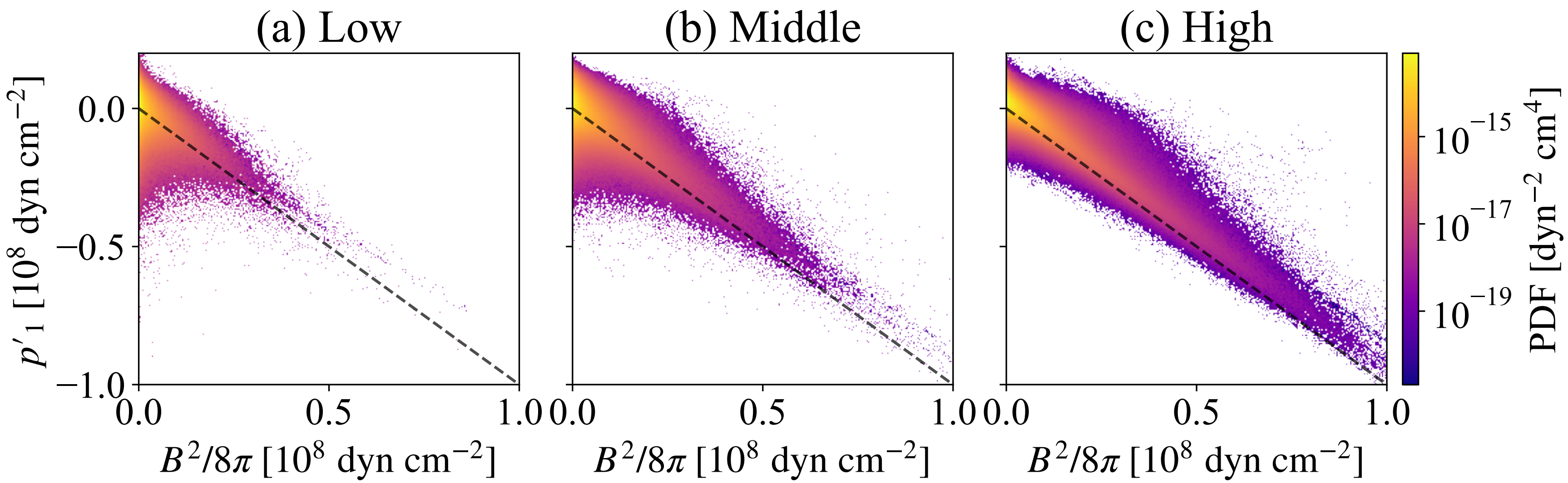


The high resolution tends to show inefficient stretching due to strong magnetic field (dashed line). The compression is increased in High case (dotted line).

The compression is the essential mechanism to construct the superequipartition magnetic field.

Magnetic field generation (2/2)

2D histogram between gas pressure perturbation p_1' and magnetic pressure $B^2/8\pi$



The dashed line indicates $p_1' = B^2/8\pi$.

In High case, the most of data are on the $p_1' = B^2/8\pi$ line. This result indicates that the strong magnetic field is maintained by the gas pressure, that is, we can use the internal energy which is massive in the Low Mach number situation in the solar convection zone.
Easy to generate the superequipartition field to the kinetic energy.

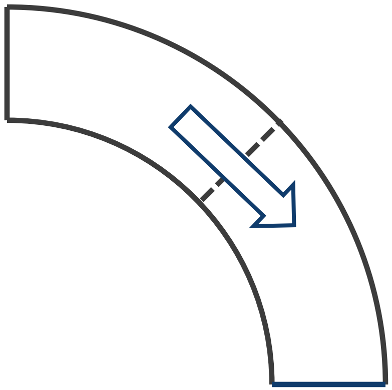
Angular momentum transport

Angular momentum conservation $\lambda = r\sin\theta,\ \mathcal{L} = \lambda v_\phi + \lambda^2\Omega_0$

$$\frac{\partial}{\partial t}\left(\rho_0\langle\mathcal{L}\rangle\right) = \underbrace{-\left(\rho_0\langle\boldsymbol{v}_\mathrm{m}\rangle\cdot\nabla\right)\langle\mathcal{L}\rangle}_{\text{Mean flow transport}} \underbrace{-\nabla\cdot\left(\lambda\rho_0\langle\boldsymbol{v}'_\mathrm{m}v'_\phi\rangle\right)}_{\text{Turbulent transport}} + \underbrace{\frac{1}{4\pi}\langle\nabla\cdot\left(\lambda\boldsymbol{B}_\mathrm{m}B_\phi\right)\rangle}_{\text{Magnetic transport}}$$

It was thought that the turbulent correlation $\langle v'_i v'_j\rangle$ has a key role to maintain the fast equator(e.g. Miesch+2000)

Latitudinal angular momentum transport



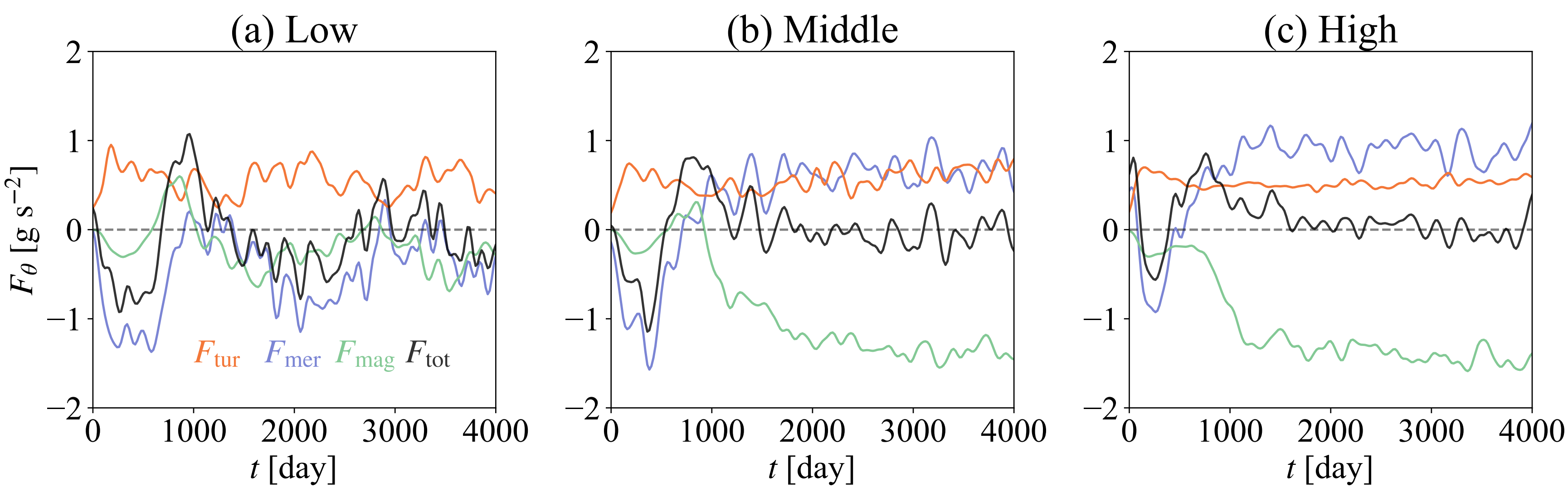
Angular momentum flux at $\theta = 45°$

Turbulent flow $$F_{\rm tur} = \rho_0 \lambda \langle v'_\theta v'_\phi \rangle,$$

Mean flow $$F_{\rm mer} = \rho_0 \langle v_\theta \rangle \langle \mathcal{L} \rangle,$$

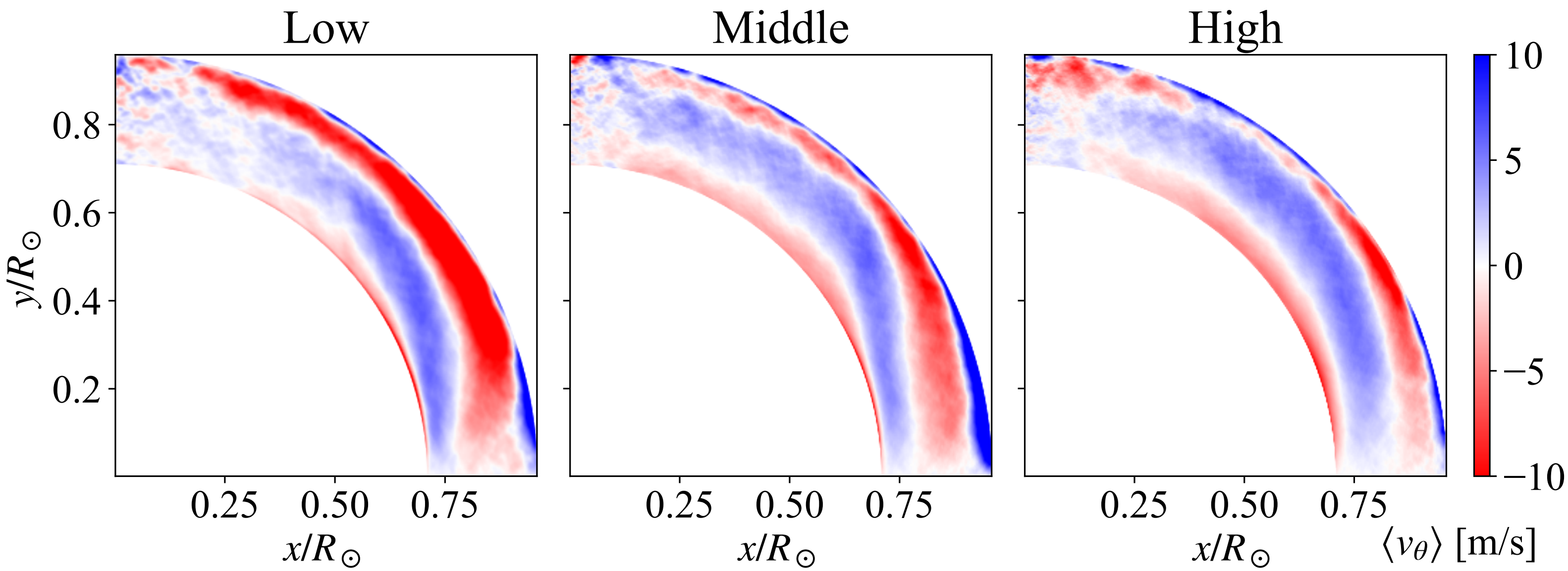
Magnetic field $$F_{\rm mag} = -\lambda \frac{\langle B_\theta B_\phi \rangle}{4\pi},$$

$$F_{\rm tot} = F_{\rm tur} + F_{\rm mer} + F_{\rm mag}.$$



Meridional flow is responsible for the fast equator

Meridional flow



The poleward flow around the base of the convection zone becomes prominent in high-resolution models.
Since the deep CZ is in a low Mach number situation, $\nabla \cdot (\rho_0 \boldsymbol{v}) = 0$ is approximately satisfied. Thus $\int \rho_0 \langle v_\theta \rangle r dr \sim 0$ at an arbitrary latitude.
The poleward meridional flow cause the equatorward meridional flow in the middle of the convection zone, which leads to net equatorward angular momentum transport.

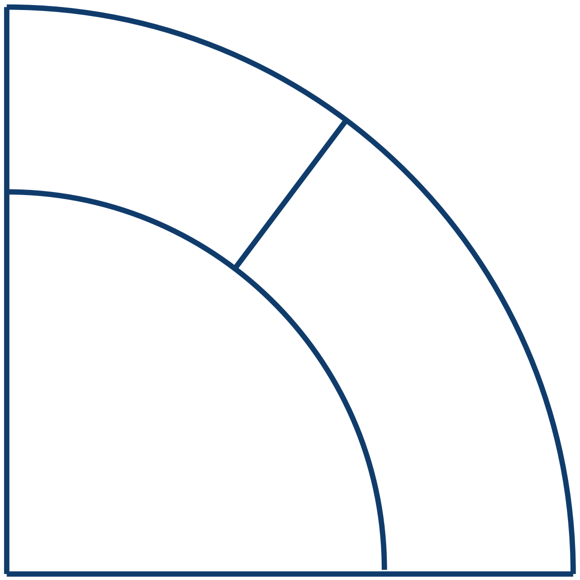
$$\mathcal{L} = r^2 \sin^2 \theta \left(\Omega_0 + \Omega_1 \right)$$

Slow rotation case (2/2)

Important slide

Anti-clockwise meridional flow always transport the AM poleward.
The colatitudinal AM by the meridional flow is:
$\rho_0\langle v_\theta\rangle\langle\mathcal{L}\rangle$, where $\mathcal{L} = r\sin\theta\, u_\phi$.

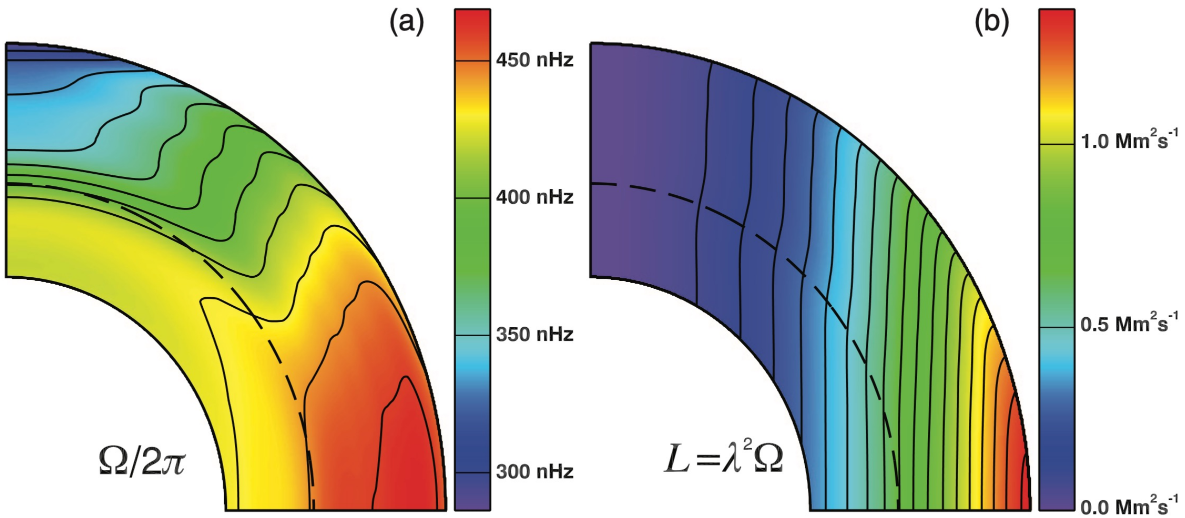
In addition, due to the extremely low Mach number the fluid satisfies the anelastic approximation $\nabla\cdot(\rho_0 \boldsymbol{v}_\mathrm{m}) = 0$, which lead to $\int \rho_0\langle v_\theta\rangle r dr = 0$ at constant θ surface.



Thus, the net AM transport $\int \rho_0\langle v_\theta\rangle\langle\mathcal{L}\rangle r dr$ is determined by the specific AM distribution, here Since the solar differential rotation is not strong $\Omega\sim\Omega_0$ and the deeper layer (small r) has smaller AM $\mathcal{L}\sim r^2\sin^2\theta\,\Omega_0$.

Thus, the anti-clockwise meridional flow MUST transports AM poleward.
We need to suppress negative $\langle v_r' v_\phi'\rangle$ to have the fast equator.

Meridional flow in quasi-steady state



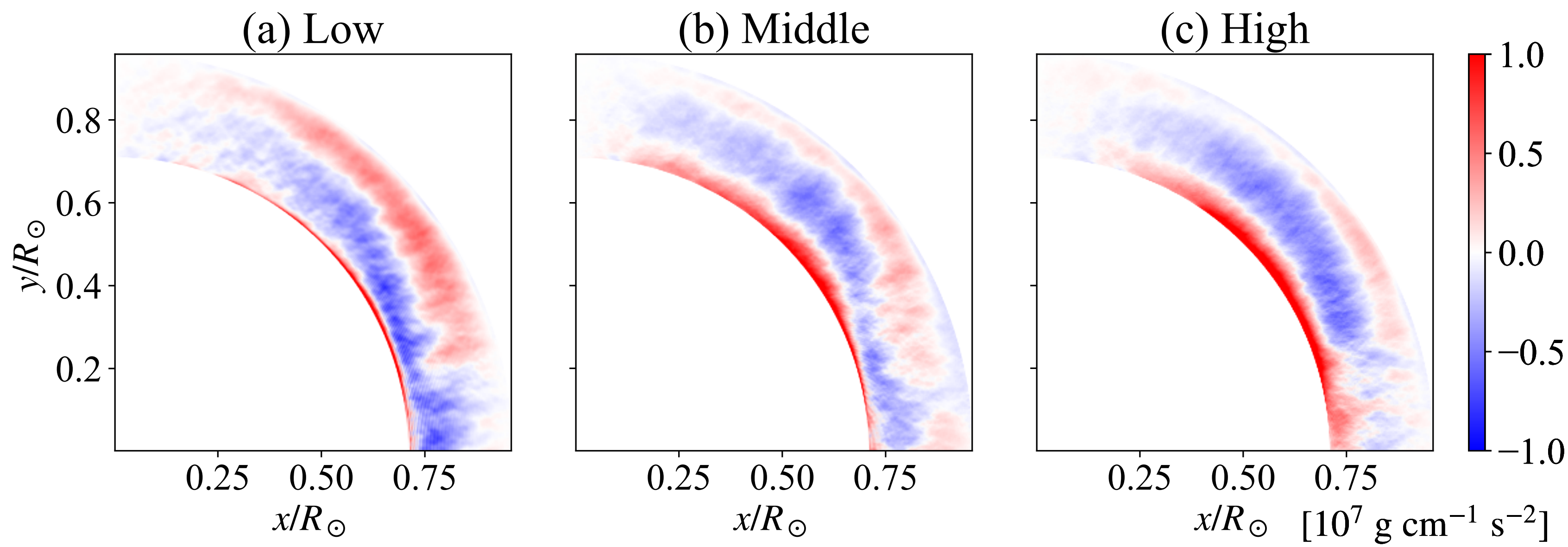
Miesch+2011

$$\left(\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \cdot \nabla\right) \langle \mathcal{L} \rangle = -\nabla \cdot \left(\lambda \rho_0 \langle \boldsymbol{v}'_\mathrm{m} v'_\phi \rangle\right) + \frac{1}{4\pi} \langle \nabla \cdot (\lambda \boldsymbol{B}_\mathrm{m} B_\phi) \rangle$$

Since the differential rotation is weak and the angular momentum does not change significantly, the topology of the meridional flow is directly determined by the Reynolds and the Maxwell stress.

Angular momentum transport by meridional flow

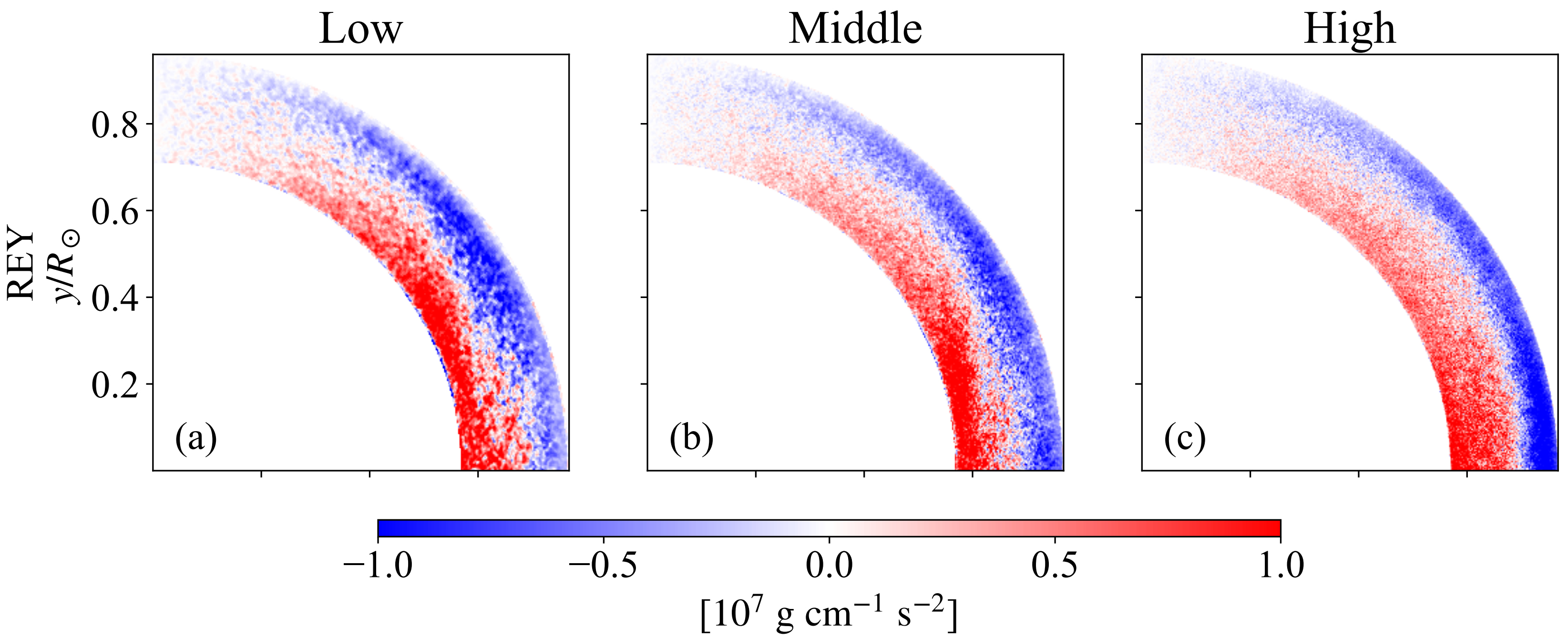
$$0 = -\left(\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \cdot \nabla\right) \langle \mathcal{L} \rangle - \nabla \cdot \left(\lambda \rho_0 \langle \boldsymbol{v}'_\mathrm{m} v'_\phi \rangle\right) + \frac{1}{4\pi} \langle \nabla \cdot (\lambda \boldsymbol{B}_\mathrm{m} B_\phi) \rangle$$



Due to the poleward meridional flow around the base of the convection zone, the angular momentum transport by meridional flow increases angular momentum there. What compensate it?

Turbulent angular momentum transport

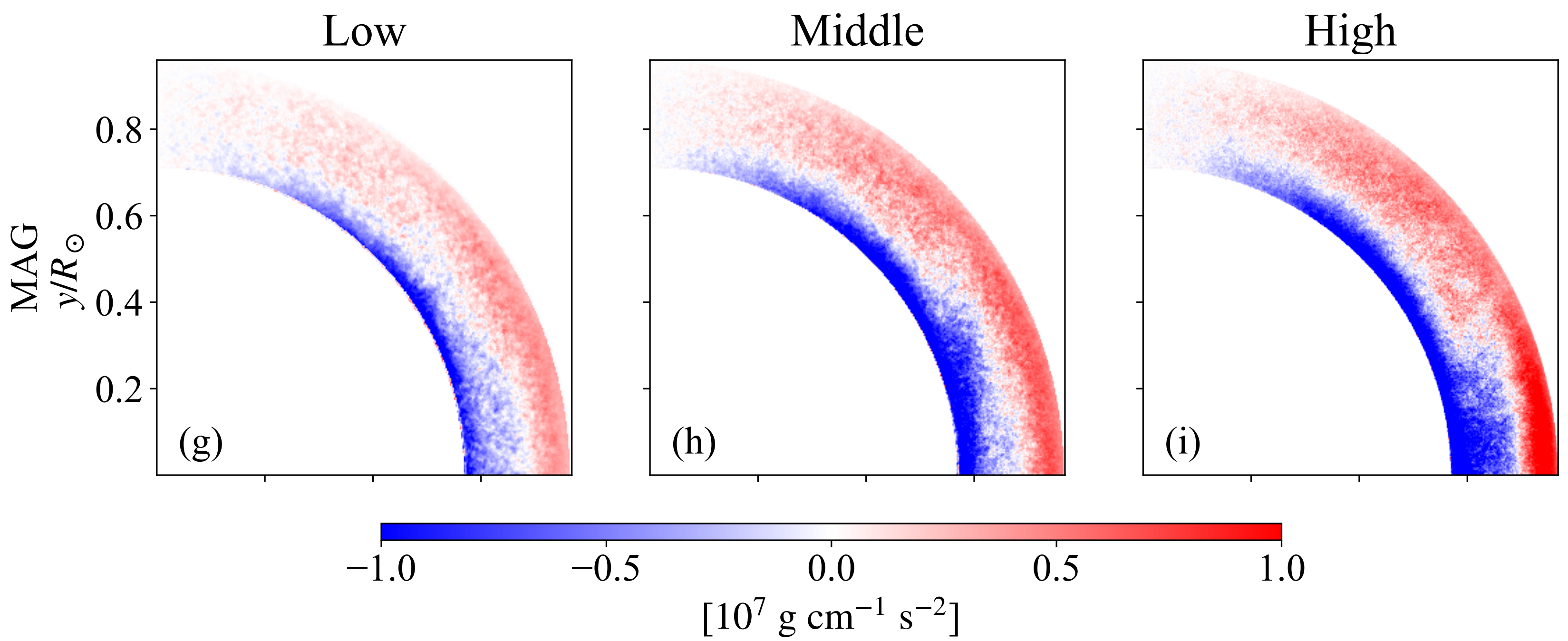
$$0 = -\left(\rho_0 \langle \boldsymbol{v}_\mathrm{m} \rangle \cdot \nabla\right) \langle \mathcal{L} \rangle - \nabla \cdot \left(\lambda \rho_0 \langle \boldsymbol{v}'_\mathrm{m} v'_\phi \rangle\right) + \frac{1}{4\pi} \langle \nabla \cdot (\lambda \boldsymbol{B}_\mathrm{m} B_\phi) \rangle$$



The turbulent angular momentum transport decelerates the equator region more in High case.
The flow does not have a role to construct the poleward meridional flow at the base

Magnetic angular momentum transport

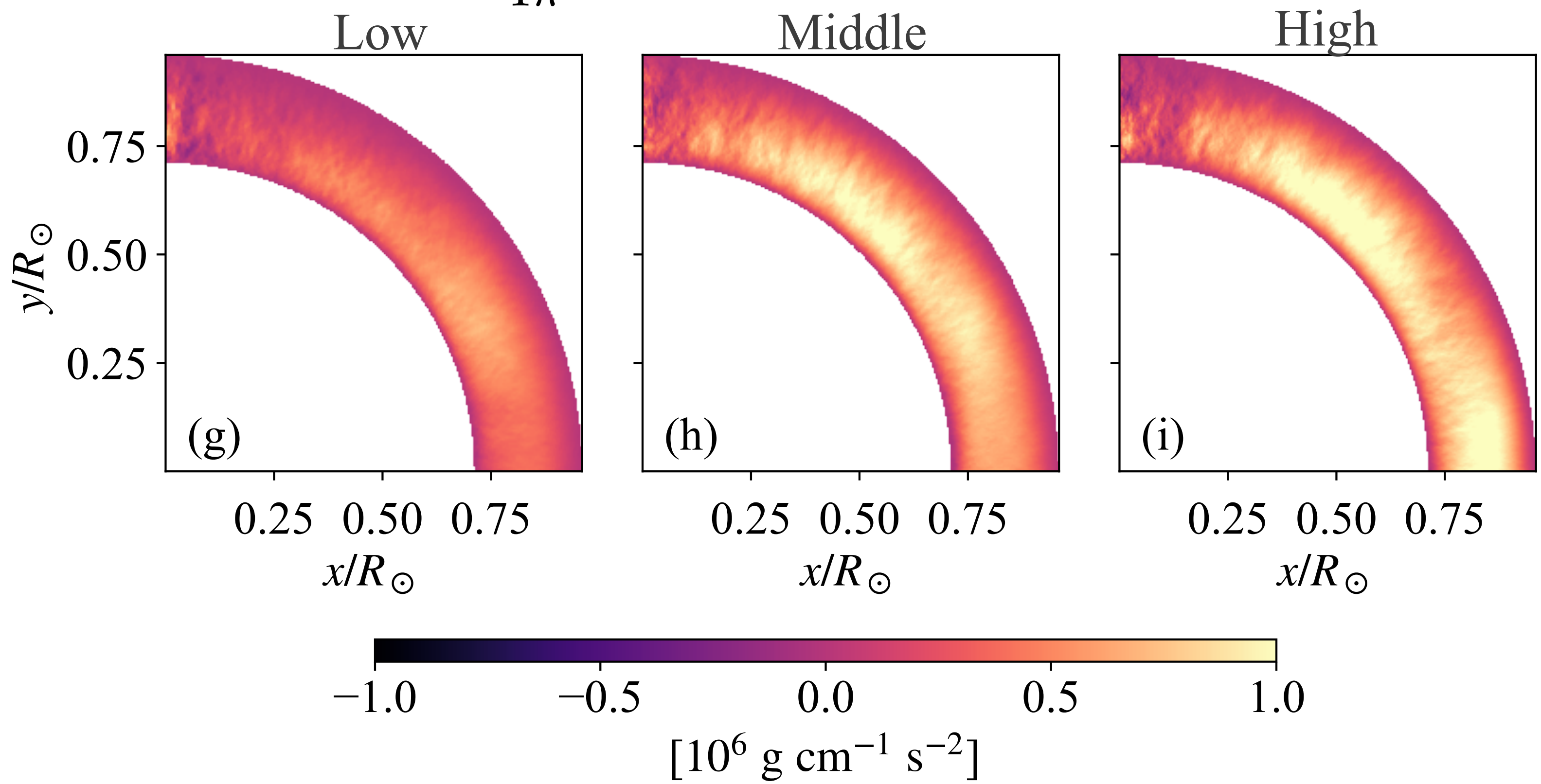
$$0 = -\left(\rho_0\langle \boldsymbol{v}_\mathrm{m}\rangle \cdot \nabla\right)\langle \mathcal{L}\rangle - \nabla\cdot\left(\lambda\rho_0\langle \boldsymbol{v}'_\mathrm{m} v'_\phi\rangle\right) + \frac{1}{4\pi}\langle \nabla\cdot\left(\lambda \boldsymbol{B}_\mathrm{m} B_\phi\right)\rangle$$



In High case, the Lorentz force has a dominant role to transport the angular momentum transport to construct the fast equator. We need to investigate the correlation $\langle B'_i B'_j\rangle$ to understand the mechanism.

Magnetic field correlation

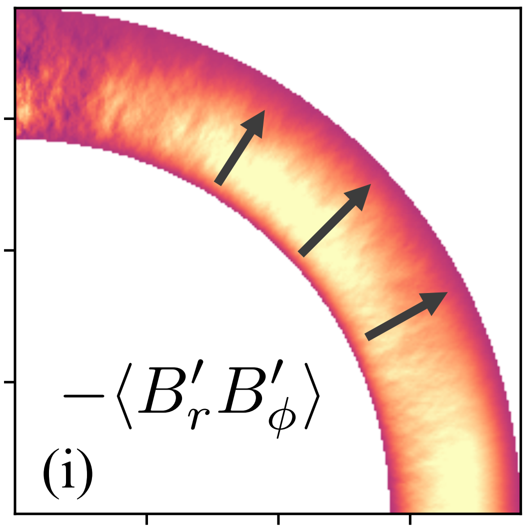
$$-\frac{\langle B_r B_\phi \rangle}{4\pi}$$: Radial momentum transport

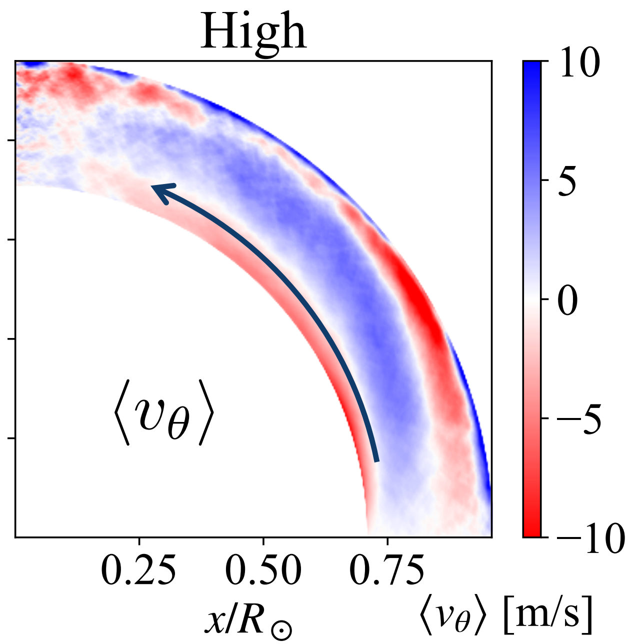


Negative correlation $\langle B_r B_\phi \rangle$ is the essential reason why we have the solar-like fast equator. The magnetic tension transport the angular momentum radially outward.

Poleward meridional flow at the base of CZ

$$\underbrace{\left(\rho_0\langle\boldsymbol{v}_\mathrm{m}\rangle\cdot\nabla\right)\langle\mathcal{L}\rangle}_{\text{meridional flow}} = \underbrace{-\nabla\cdot\left(\rho_0\lambda\langle\boldsymbol{v}'_\mathrm{m}v'_\phi\rangle\right)}_{\text{turbulence}} \underbrace{-\nabla\cdot\left(-\frac{\lambda}{4\pi}\langle\boldsymbol{B}_\mathrm{m}B_\phi\rangle\right)}_{\text{magnetic field}}$$





The poleward meridional flow at the base of CZ is caused by the radially outward angular momentum transport by the magnetic field.

Origin of negative $\langle B'_r B'_\phi \rangle$ (1/2)

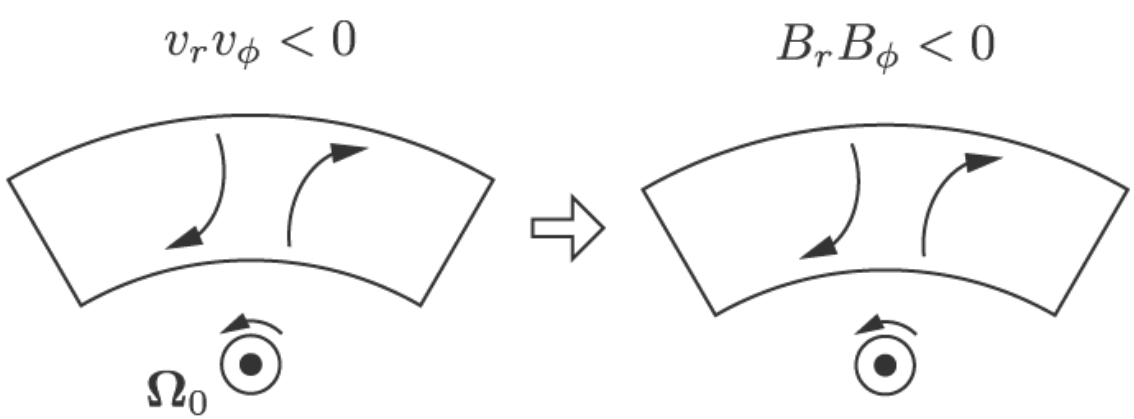
There are two major reason to create magnetic field correlation

1. Flow Shear

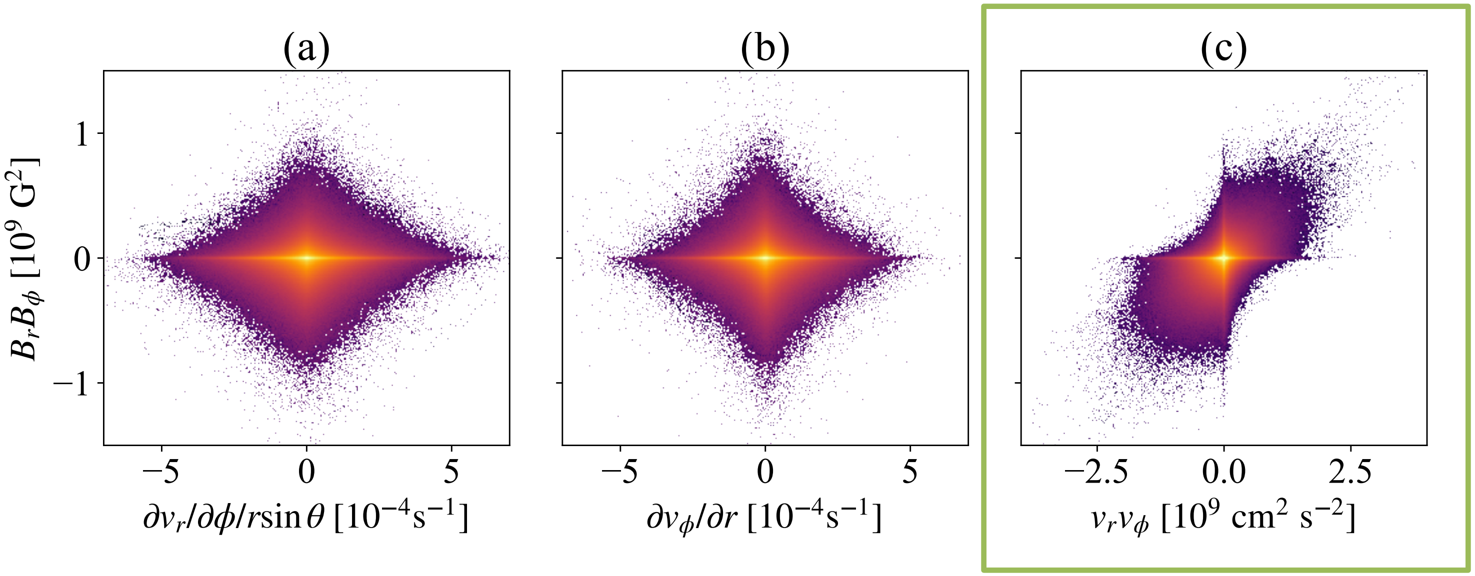
$$\frac{\partial B_r}{\partial t} = B_\phi \frac{1}{r\sin\theta}\boxed{\frac{\partial v_r}{\partial \phi}} + [\ldots], \quad \frac{\partial B_\phi}{\partial t} = B_r \boxed{\frac{\partial v_\phi}{\partial r}} + [\ldots]$$

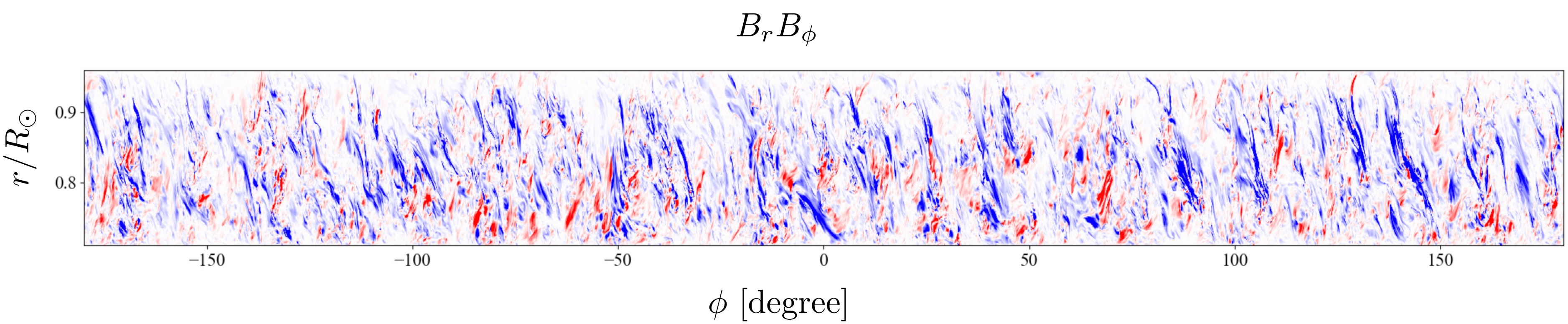
2. Magnetic field tends to be parallel to flow. $\boldsymbol{v}\times\boldsymbol{B} = 0$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$



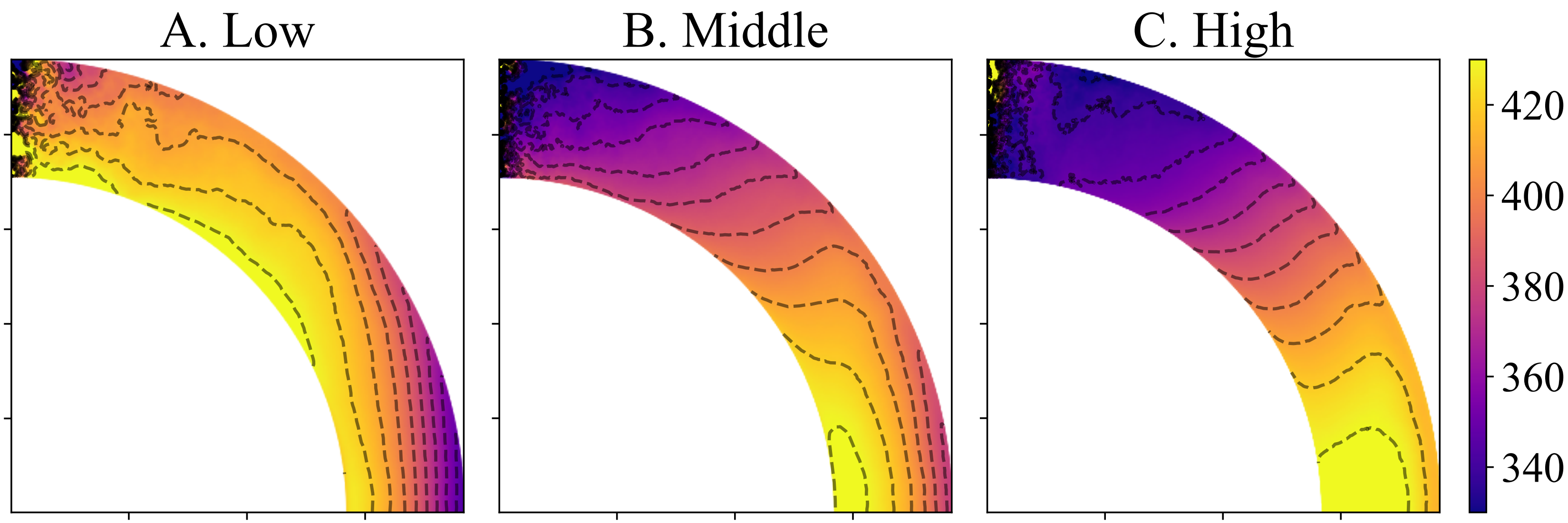
Origin of negative $\langle B'_r B'_\phi \rangle$ (2/2)





The magnetic field is actually inclined.

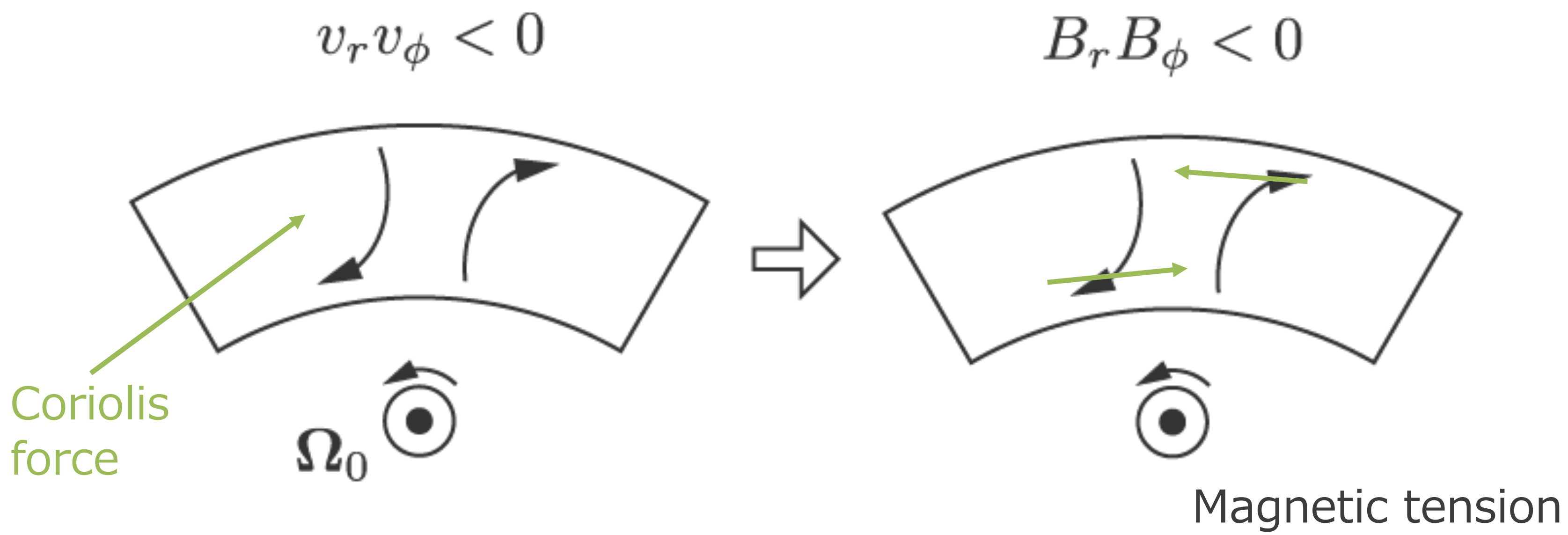
Additional role of magnetic field



A prominent difference between the Middle and High cases are the angular velocity at the near surface equator.

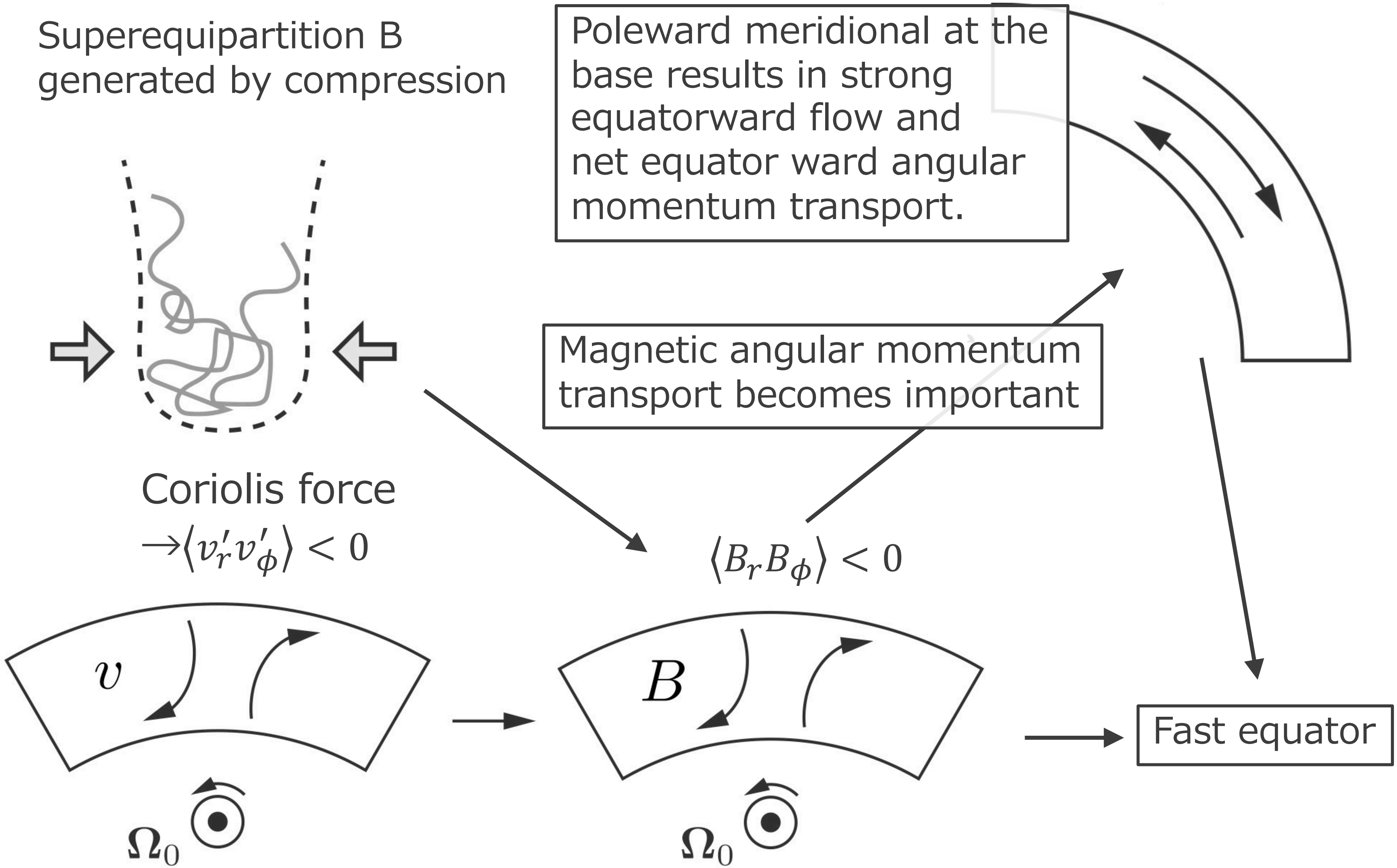
The radially outward angular momentum transport by the magnetic field helps construction

Mechanism for the radially outward angular momentum transport

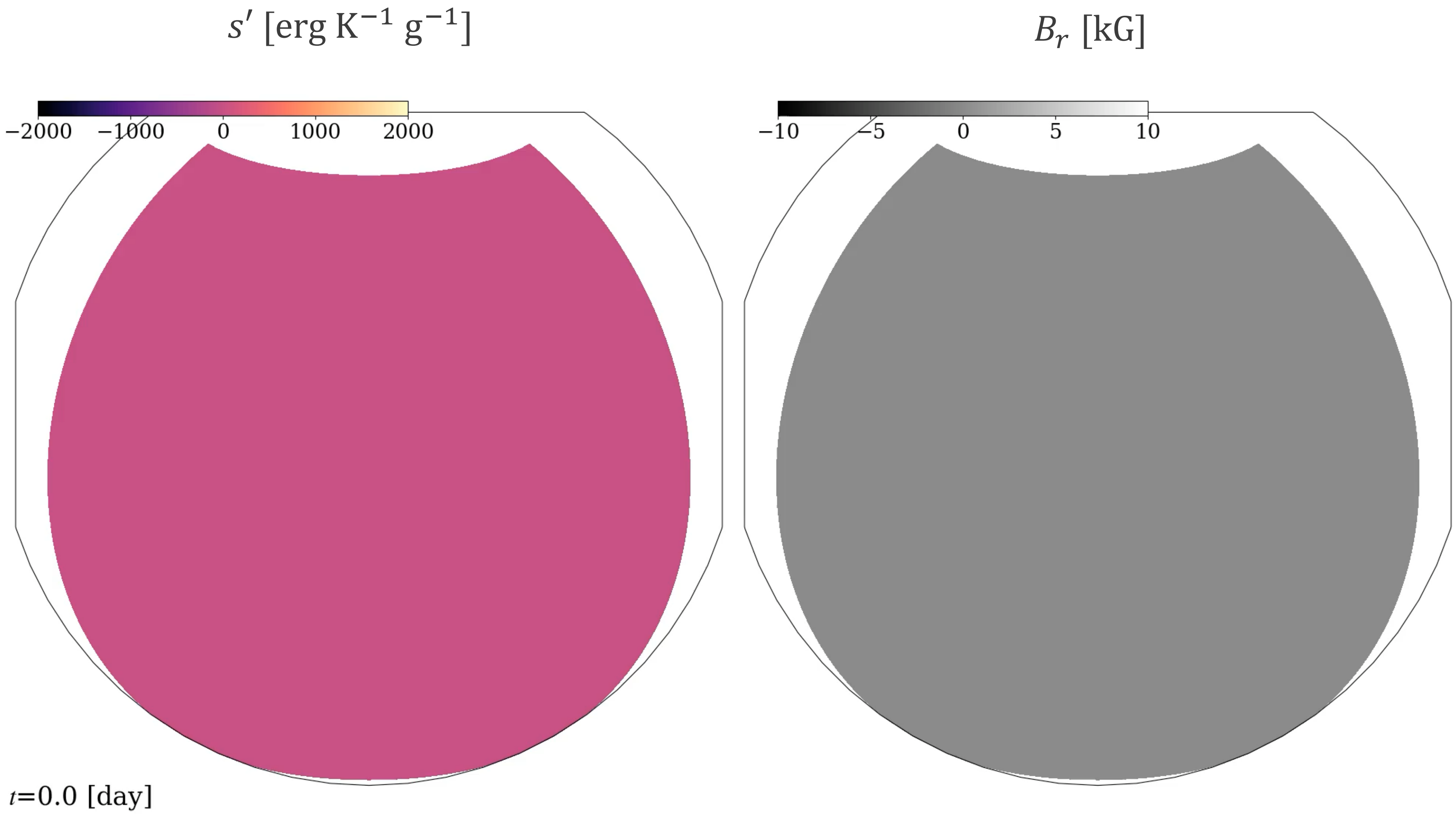


1. Coriolis force cause negative correlation $\langle v_r v_\phi \rangle$, inclined flow.
2. The magnetic field tends to be parallel to the flow and also inclined.
3. The negative correlation $\langle B_r B_\phi \rangle$, which transports angular momentum radially outward via the magnetic tension.

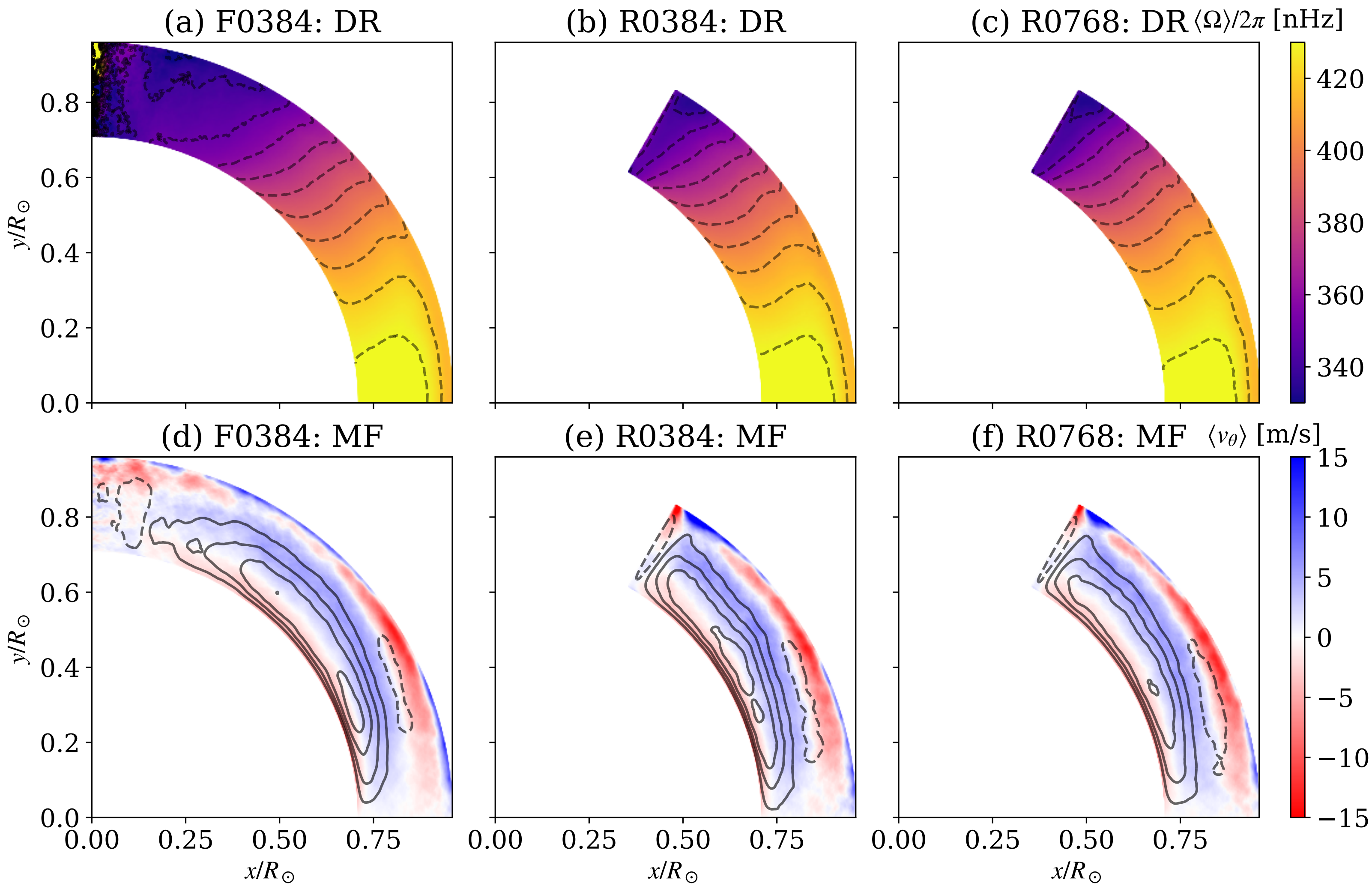
The whole story



Numerical convergence: 1024×6144×6144 20 M time step



Numerical convergence: Differential rotation and meridional flow



We hardly see further resolution dependence. Numerical convergence?

Summary

- ✓ We carry out super-high resolution simulation for the solar convection zone (5.4 billion grid points)
- ✓ The solar-like differential rotation is reproduced without using any manipulation
- ✓ The magnetic energy is much larger than kinetic energy

- ✓ The strong magnetic field is maintained by the internal energy
- ✓ The angular momentum is transported by the magnetic field
- ✓ The magnetic correlation is originated by the Coriolis force.
  → We call this process "Punching ball effect".